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Lepton Flavour Violation in the extensions of the Standard Model

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Abstract

In this doctoral thesis, we investigate lepton CP- and flavor-violating observables in the general extension of the Standard Model (SM), where possible New Physics effects are parameterized by the dimension-5 and -6 gauge invariant operators constructed from the SM fields. We consider only the observables related to the charged lepton sector, as in the SM (extended with massive neutrino sector only) such processes are very strongly suppressed and thus any experimental confirmation of charged lepton flavor violation would immediately signal some phenomena going beyond the SM.

We focus on the effective description of the SM extensions in terms of higher order effective operators and calculate the charged lepton CP- and flavor-related observables as a functions of Wilson coefficients multiplying such operators. Then, using the experimental results, we put bounds on those coefficients. Such a model independent approach facilitates analysis of specific New Physics theories - only the values of Wilson coefficients need to be calculated within any such theory and then compared with the bounds obtained in this thesis.

In particular, we calculate the decay rates for the radiative lepton flavor violating decays $\ell_i \to \ell_f \gamma$ as well as closely related charged lepton Electric Dipole Moments and Anomalous Magnetic Moments. We also study three-body charged lepton flavor violating decays $\ell_i \to \ell_j \ell_k \ell_l$ and Z-boson decays into pair of leptons of different flavors. For each process, we present compact final formulae in terms of Wilson coefficients of subset of operators contributing to given process, in the form allowing easy comparison to experimental measurements. We also discuss possible numerical correlations between the constraints put by experiments on different coefficients.

The results of this thesis are published in [1] and were presented at the Moriond 2013 and 2014 conferences [2, 3].

Contents

Chapter 1

Introduction

The Standard Model of strong and electroweak interaction (SM) is a theory that successfully explains and predicts the elementary particle phenomenology at the energy scales currently reachable in the experiments. However, it is often considered to be only an effective approximation of some more fundamental theory, valid at much higher energy scales.

A hint on how this fundamental theory may look like can come from the flavor structure of the SM. So far we don't have any definite idea about the origin of the mass of quarks and leptons or flavor mixing. The flavor physics is very important as a test to search for some New Physics (NP) beyond the SM where we can understand the origin of flavors more deeply. Depending on the specific realization of New Physics, various patterns of the relative size of amplitudes of the flavor violating decays can emerge. The current experiments already place strong constraints on many models beyond the SM.

In the SM, the Cabibbo-Kobayashi-Maskawa (CKM) matrix describes the flavor violation between the three different families of quarks. In current experiments searching for the charged lepton properties, the lepton number of each family (lepton flavor) is conserved. On the other hand, the discovery of flavor violation or neutrino oscillations in the SNO (solar neutrino) [4], SuperKamiokande (atmospheric neutrinos) [5], KamLAND (reactor neutrinos) [6], MINOS (accelerator neutrinos) [7] and other neutrino experiments provides an experimental observation of Lepton Flavor Violation (LFV) and a possible hint of new physics beyond the Standard Model. So far neutrino oscillation is the only LFV effect that has been observed.

The masses of the neutrinos are a puzzle in the SM. There is not yet known a unique way how to extend the SM to describe neutrino masses. One possibility is to add the right handed neutrino sector to the SM. Then the mass of the neutrinos can be generated exactly in the same way as the masses of the charged fermions are generated by the Higgs couplings after the electroweak (EW) symmetry breaking. In such case, the flavor mixing in lepton sector would be generated in the same way as the quark sector. However, as right neutrinos have vanishing SM gauge quantum numbers, their phenomenology can be more complicated and include so called Majorana mass terms.

Irrespective of the origin of neutrino masses, in principle they can give rise to flavor changing radiative charged lepton decays, $\mu \to e\gamma$, $\tau \to e\gamma$ and $\tau \to \mu\gamma$, and also to the 3-body charged lepton decay such as $\mu \to eee$, which are forbidden in the SM with massless neutrinos. However, in the extended SM with the neutrino mass terms only, the GIM mechanism makes the respective branching ratios (BR) of these decays very small due to the smallness of the mass of the neutrino comparing to the mass of W boson, the heaviest particle in the loop. The prediction for the branching ratios for such decays are of the order of 10^{-56}, too small to observe at experiments. Thus, LFV in the charged lepton sector could be observed only in New Physics would manifest itself also in some other ways.

Experimentally, the charged lepton flavor violating decays have never been yet observed. But as there are many models beyond the SM such as supersymmetry or other theories (technicolor, extra dimensions etc.) which predict sizable LFV rates, not far from the current experimental bounds, various experiments actively search for such phenomena. Belle detector at the KEKB e^+e^-collider, is searching for $\tau \to \mu\gamma$ and $\tau \to e\gamma$. The new upper limits on the branching ratios are $B(\tau \to \mu\gamma) < 4.5 \times 10^{-8}$ and $B(\tau \to e\gamma) < 4.5 \times 10^{-7}$. The Belle experiment is also in operation to search for lepton flavor violating τ decays into three leptons, the latest upper limits for the branching ratios are about 10^{-8}. The MEG (Mu to Electron and Gamma) experiment at Paul Scherrer Institute (PSI) in Switzerland tries to measure the $(\mu \to e\gamma)$ decay since 2009. The latest bound for this decay rate is $BR(\mu \to e\gamma) < 5.7 \times 10^{-13}$ at 90% $C.L.$ For the decay of $\mu^+ \to e^+e^-e^+$ the upper limit obtained by SINDRUM experiment is 1.0×10^{-12}. Mu3e experiment at PSI is going to search for the decay $\mu^+ \to e^+e^-e^+$. This experiment will be performed in two phases. In the first phase (2014-2017), it would provide a sensitivity of BR($\mu^+ \to e^+e^-e^+$)$\approx 10^{-15}$. In the second phase, the experiment will reach the sensitivity of BR($\mu^+ \to e^+e^-e^+$)$\approx 10^{-16}$.

Apart from flavor transitions, other experiments search also for the signs of CP violation, as it may be related to the creation of the baryon asymmetry of the universe. In the Kaon and B meson sector CP violation has been observed and can be explained within the SM as the 3×3 quark mixing (the CKM matrix) can accommodate a CP violating phase. In the lepton sector no CP violation has been detected yet, and the most accurate and promising searches concentrate on the measurements of Electric Dipole Moments (EDM), as only if CP symmetry is violated, elementary particles can possess EDMs. However, the maximum possible values of EDMs in the context of the SM are again too small to be detected experimentally. In the SM all EDMs vanish exactly at two loop level and tree loop contributions have been evaluated to obtain some non-trivial result. The SM predict a non-zero electron EDM on the order of $10^{-38} e.cm$. So far, no EDM of any particle has been observed but there are many models beyond the SM that the value of EDMs is large enough to be detected at experiments in a not very distant future.

Another hint for physics beyond the SM can come from the CP-conserving but closely related to EDMs (both come from form factors of the effective fermion-photon vertex) magnetic dipole moments of a particles. The magnetic dipole moments are proportional to the intrinsic spin of particles, with their relative coefficient known as the gyromagnetic ratio. The Dirac equation predicts that the gyromagnetic for the charged spin 1/2 fermions is exactly 2. Loop corrections from quantum field theory interactions to this tree level prediction has been calculated within the SM with great accuracy, reaching 11 significant digits in case of electron. Experimental measurements of $g-2$ value are also very accurate and in the case of muon magnetic moment show actually a discrepancy at the level of 3σ with the SM expectation, one of the very few such deviations known currently. The new project aims to measure the muon anomalous magnetic moment in the $g-2$ BNL E821 experiment and is on it's way to Fermilab for run around 2016.

Finally, there many models beyond the SM which predict also the LFV Z boson decay to lepton pairs at a range that is possible to reach at experiments (although in the SM with massive neutrino it is again too small and non observable) Thus, it is another interesting place to search for NP. The current best experimental bounds (from LEP1 experiments) on the $Z \to ll'$ branching ratios are the order of 10^{-6}.

As said above, the renormalizable SM is probably an effective theory valid only up to some energy scale Λ and a more fundamental theory can manifest itself above that scale. One way to investigate physics beyond the SM is to calculate LFV observables in a given specific New Physics model. Then one can constraint parameters of the model by comparing its predictions with experimental bounds. In this way we have to do full calculation of interesting processes for each model separately, starting from the the Lagrangian defining the NP theory. Such a procedure is usually time and labor consuming and contain some "redundant" parts, i.e. the ones which repeat from model to model.

Another approach, that we are interested in this thesis, is at least partially model independent. At the electroweak scale the effects of NP can be parametrized by some non-standard interactions described by non-renormalizable operators of higher mass dimensions. Such operators should be constructed out of the SM fields and preserve its gauge symmetry and the pattern of spontaneously symmetry breaking (SSB). Coefficients of new higher dimension operators (called further Wilson coefficients) are suppressed by heavy mass scale at which New Physics should became effective. They make the extended Lagrangian non-renormalizable. In general the dimension-4 renormalizable SM Lagrangian can be extended as follows:

$$\mathcal{L}_{\rm ESM} = \mathcal{L}_{\rm SM} + \mathcal{L}_{\rm d=5} + \mathcal{L}_{\rm d=6} + \ldots . \tag{1.1}$$

Using such Lagrangian, the theoretical calculations of relevant physical observables can be performed already in a model-independent way, such that final formulae are given in terms of "generic" Wilson coefficients. Having such expressions simplifies significantly a comparison of various SM extension with the experimental results, as now only the values Wilson coefficients of new operators need be calculated within a given model of NP - this part of analysis is always model-dependent.

In this thesis we follow that reasoning, calculating the LFV processes described earlier in this Chapter in terms of coefficients of higher dimension operators. In general, as discussed in [8], there is only one dimension-5 gauge invariant operator (describing interaction of two lepton doublets and two Higgs doublets generates Majorana mass terms for neutrinos), but there are 59 independent dimension-6. Fortunately, as discussed in more details in Chapter 3, only much less numerous subset of them is important for the LFV processes in the charged lepton sector, so our numerical analysis is quite predictive.

This dissertation has the following content. In the introductory Chapter 2, we present the SM of particle physics extended with massive neutrino sector (νSM). We summarize there the νSM predictions for radiative and tree body charged lepton decays. In the following Chapters we discuss the lepton flavor violation in the extended SM with general gauge invariant dimension-6 operators. In Chapter 3, we describe in details how to parametrize the extensions of the SM with higher dimension operators. In Chapter 4, we discuss the radiative lepton decay as well as closely electric dipole moment and anomalous magnetic moment of the charged leptons. Then in Chapter 5 we present our analysis of the three body charged lepton decay. Finally in Chapter 6 we consider the possibility of Z boson decays to lepton pair with flavor violation. In the last Chapter 7 we present the conclusions derived from our study of the lepton flavor violating in the extended SM with dimension-6 operators. The thesis contains also three Appendices, containing Feynman rules for the leptonic interactions of the extended SM, definitions and expansions of 2- and 3-point loop integrals and an example with detailed calculation of some of the diagrams contributing to effective lepton-photon vertex. Last part of the thesis contains the bibliography of the subject.

Chapter 2

Lepton Flavor Violation in the Standard Model with massive neutrinos

We start our consideration from discussion of the LFV processes in Standard Model extended with massive neutrino sector. We recall the SM Lagrangian, with emphasis on its leptonic part, and summarize the SM predictions for the most important LFV processes in the charged lepton sector.

2.1 The SM Lagrangian

	fermions					scalars
field	ℓ_{Li}^a	e_{Ri}	q_{Li}^a	u_{Ri}	d_{Ri}	φ^a
hypercharge Y	$-\frac{1}{2}$	-1	$\frac{1}{6}$	$\frac{2}{3}$	$-\frac{1}{3}$	$\frac{1}{2}$

Table 2.1: The SM matter content and filed hypercharges.

The Standard Model(SM) is a gauge theory based on the gauge symmetry group $SU(3)\times SU(2)\times U(1)$ with the following field content:

- Matter fermions
  - Left-handed lepton doublets: l_p^i

- Right-handed charged leptons: e_p
- Left-handed quark doublets: $q_p^{\alpha i}$
- Right-handed quarks: u_p^α, d_p^α

• Gauge fields

- $SU(3)_c$ bosons (gluons): G_μ^A, $A = 1, \ldots, 8$
- $SU(2)$ bosons: W_μ^I, $I = 1, 2, 3$
- $U(1)$ boson B_μ

• Higgs boson doublet: φ^i (we denote also $\widetilde{\varphi}^i = \varepsilon^{ij}\varphi_j^\star$)

where α, i, and p are respectively, color, weak isospin and flavor indices. The SM matter content is summarized in Table 2.1, where also the $U(1)$ hypercharges Y are given.

The SM Lagrangian can be split into several parts:

$$\mathcal{L} \quad = \quad \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{Higgs}} + \mathcal{L}_{\text{Yukawa}}. \tag{2.1}$$

The Lagrangian for the gauge interaction is given by:

$$\mathcal{L}_{gauge} \quad = \quad -\frac{1}{4}G^A_{\mu\nu}G^{A\mu\nu} - \frac{1}{4}W^I_{\mu\nu}W^{I\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} + (D_\mu\varphi)^\dagger(D^\mu\varphi), \tag{2.2}$$

with the field strength tensors and covariant derivatives defined as

$$\begin{aligned} G^A_{\mu\nu} &= \partial_\mu G^A_\nu - \partial_\nu G^A_\mu + g_s f_{ABC} G^B_\mu G^c_\nu, \\ W^I_{\mu\nu} &= \partial_\mu W^I_\nu - \partial_\nu W^I_\mu + g, \varepsilon_{IJK} W^J_\mu W^K_\nu \\ B_{\mu\nu} &= \partial_\mu B_\mu - \partial_\nu B_\mu, \end{aligned} \tag{2.3}$$

$$D_\mu = \partial_\mu - ig_s\frac{1}{2}\lambda^A G^A_\mu - ig\frac{1}{2}\tau^I W^I_\mu - ig'YB_\mu. \tag{2.4}$$

f_{ABC} and ε_{IJK} are structure constants for $SU(3)$ and $SU(2)$ groups, respectively, λ^A and τ^I are their generators in triplet and doublet representations and g_s, g, g' are coupling constants for $SU(3)$, $SU(2)$ and $U(1)$ respectively.

The Higgs Lagrangian is given by

$$\mathcal{L}_{Higgs} \quad = \quad m^2\varphi^\dagger\varphi - \frac{\lambda}{2}(\varphi^\dagger\varphi)^2, \tag{2.5}$$

where m is the Higgs mass parameter and λ is the Higgs coupling parameter.

All particles obtain their mass due to the spontaneous breaking of $SU(2_L)$ symmetry group via a non vanishing vacuum expectation value of the Higgs field. As the result, the gauge group of the SM is spontaneously broken into $SU(3)_c \times U(1)_{EW}$. The $SU(3)$ gauge bosons, the gluons, remain massless. The physical intermediate weak bosons combinations are given by:

$$W^{\pm}_{\mu} = \frac{W^1_{\mu} \mp iW^2_{\mu}}{\sqrt{2}}, \qquad Z_{\mu} = - \sin\theta_W B_{\mu} + \cos\theta_W W^3_{\mu}, \tag{2.6}$$

and their masses are ($\tan\theta_W = \frac{g'}{g}$):

$$m_W = \frac{1}{2} g v, \quad m_Z = \frac{m_W}{\cos\theta_W}, \tag{2.7}$$

The photon field defined as:

$$A_{\mu} = \cos\theta_W B_{\mu} + \sin\theta_W W^3_{\mu}, \tag{2.8}$$

remains massless.

The Yukawa Lagrangian is given by:

$$\mathcal{L}_{Yukawa} = -\Gamma_e \bar{l} e \varphi - \Gamma_u \bar{q} u \widetilde{\varphi} - \Gamma_d \bar{q} d \varphi + H.c. \tag{2.9}$$

Where Γ_e, Γ_d and Γ_u are 3×3 Yukawa coupling matrices for the charged leptons, the down-type quarks and up-type quarks respectively.

Summarizing, the general form of the Lagrangian of the SM based on the gauge group $SU(3) \times SU(3) \times U(1)$ can be written down as:

$$\begin{aligned} \mathcal{L}_{\mathrm{SM}} = & - \frac{1}{4} G^A_{\mu\nu} G^{A\mu\nu} - \frac{1}{4} W^I_{\mu\nu} W^{I\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} \\ & + (D_{\mu}\varphi)^{\dagger}(D^{\mu}\varphi) + m^2 \varphi^{\dagger}\varphi - \frac{\lambda}{2}(\varphi^{\dagger}\varphi)^2 \\ & + i(\bar{l}\not{D}/l + \bar{e}\not{D}e + \bar{q}\not{D}q + \bar{u}\not{D}u + \bar{d}\not{D}d) \\ & - (\Gamma_e \bar{l} e \varphi + \Gamma_u \bar{q} u \tilde{\varphi} + \Gamma_d \bar{q} d \varphi) + H.c. \end{aligned} \tag{2.10}$$

2.2 Fermion masses and mixing

The SM fermions can be grouped into so-called generations, differing by the quantum number known as flavor. E.g. in the lepton sector one can assign the electron number L_e ($L_e = 1$ for e^-, ν_e and $L_e = -1$ for $e^+, \bar{\nu}_e$), the muon number L_μ ($L_\mu = 1$ for μ^-, ν_μ and $L_\mu = -1$ for

$\mu^+, \bar{\nu_\mu}$) and the τ number L_τ ($L_\tau = 1$ for τ^-, ν_τ and $L_\tau = -1$ for $\tau^+, \bar{\nu_\tau}$). Then the three families of leptons

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}, \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}, \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}$$

each have definite lepton number, and similarly in the quark sector.

The Yukawa part of the Lagrangian couple the left-handed Weyl fermion doublets of $SU(2)_L$ with the right-handed Weyl fermion singlets and the Higgs boson and mix the fermions from different generations. By substituting the vacuum expectation value for the Higgs field, the Yukawa interaction generates fermion mass terms, proportional to the Yukawa matrices:

$$\mathcal{L}_{mass} = -\frac{v}{\sqrt{2}}\Gamma_e^{\alpha\beta}\bar{e}_\alpha e_\beta - \frac{v}{\sqrt{2}}\Gamma_u^{\alpha\beta}\bar{u}_\alpha u_\beta - \frac{v}{\sqrt{2}}\Gamma_d^{\alpha\beta}\bar{d}_\alpha d_\beta. \tag{2.11}$$

The charged fermion mass matrices are:

$$\begin{aligned} [M_e] &= \frac{v}{\sqrt{2}}[\Gamma_e], \\ [M_u] &= \frac{v}{\sqrt{2}}[\Gamma_u], \\ [M_d] &= \frac{v}{\sqrt{2}}[\Gamma_d]. \end{aligned} \tag{2.12}$$

The fermion mass matrices can be general complex 3×3 matrices. Each such mass matrix can be diagonalized by two unitary transformations in the flavor space, one for the left-handed fermions and one for the right-handed fermions with the same charge. Since the unitary matrices for the left-handed up and down quarks are generally different, flavor mixing is induced in the charged weak-current (CC) interaction of quarks. The full CC interaction term is given by:

$$\begin{aligned} \mathcal{L}_{W\bar{l}l} = & -\frac{g}{\sqrt{2}}[\bar{u}_{iL}\gamma^\mu (V_{CKM})_{ij} d_{jL} W_\mu^+ + \bar{d}_{iL}\gamma^\mu (V_{CKM})^\dagger_{ji} u_{jL} W_\mu^- \\ & -\bar{\nu}_{iL}\gamma^\mu e_{iL} W_\mu^+ - \bar{e_{iL}}\gamma^\mu \nu_{iL} W_\mu^-]. \end{aligned} \tag{2.13}$$

V_{CKM} is called the Cabbibo-Kobayashi-Maskawa (CKM) matrix. In eq. (2.11) neutrinos are massless, therefore the neutrino fields can always be rotated in such a way that there is no lepton mixing in CC.

In the minimal version of SM where neutrinos are massless, lepton flavor is conserved. However, the observation of neutrino oscillation indicate neutrinos are massive. So, their mass matrix must be non-diagonal (and in general complex), as in the case of quark sector. However, as already mentioned in the Introduction, there is no common agreement yet how the neutrino masses and mixing should be added to the SM Lagrangian (most commonly considered option is adding right-handed heavy neutrinos and so called see-saw mechanism). Thus, we do not consider here any specific mechanism and just assume that the neutrino mass eigenstates are different from the flavor eigenstates:

$$\mid \nu_\alpha >= \sum_i U_{\alpha i} \mid \nu_i >, \tag{2.14}$$

where $\nu_\alpha = \nu_e, \nu_\mu, \nu_\tau$ are flavor (weak) eigenstates and $\nu_i = \nu_1, \nu_2, \nu_3$ are mass eigenstates with masses m_1, m_2, m_3. U is a unitary matrix known as Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix. The matrix can be written as follows :

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}. \tag{2.15}$$

The PMNS matrix is most commonly parametrized by the three mixing angles $(\theta_{12}, \theta_{23}, \theta_{13})$ and a single CP violation phase. If neutrinos are Majorana type one has also the 2 additional phase factors α_1 and α_2.

$$U = \begin{pmatrix} c_{12}c_{13} & \delta_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} e^{\frac{i\alpha_1}{2}} & 0 & 0 \\ 0 & e^{\frac{i\alpha_2}{2}} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{2.16}$$

where $c_{ij} \equiv \cos\theta_{ij}$ and $s_{ij} \equiv \sin\theta_{ij}$. A neutrino of flavor α with momentum p, at $t = 0$ evolves after a time interval of t as

$$\mid \nu_\alpha(t) >= e^{-iHt} \mid \nu_\alpha >= \sum_i U_{\alpha i} e^{-E_i t} \mid \nu_i(0) >, \tag{2.17}$$

where $E_i = \sqrt{p^2 + m_i^2}$. Since $m_i << p$, by using ultra relativistic approximation we have $E_i = p + \frac{m_i^2}{2p}$. The probability of finding flavor ν_β in ν_α beam at distance x from the source is given by

$$P_{\nu_\alpha \to \nu_\beta} = \sum_i \mid U_{\alpha i} \mid^2 \mid U_{\beta i} \mid^2 + \sum_{i \neq j} U_{\alpha i} U_{\beta i}^* U_{\beta i}^* U_{\alpha j}^* U_{\beta j} \cos(\frac{2\pi x}{L_{ij}}), \tag{2.18}$$

where $L_{ij} = \frac{2\pi}{(E_i - E_j)} \simeq \frac{4\pi p}{|m_i^2 - m_j^2|}$ is the oscillation length. For neutrino oscillation, non-zero neutrino mass and mixing angles are needed. Thus, experimental confirmation of such oscillations inevitably leads to the conclusion that the SM should be extended at least by new terms or interactions generating non-vanishing neutrino masses.

Such masses would contribute to the transitions between charged leptons at 1-loop level through diagrams shown in the next Section in Fig. 2.2. However the LFV process in charged lepton sector are strongly suppressed because of the GIM mechanism. Thus an observation of $l \to e\gamma$ (and also $l \to l'l'l'$ decays discussed later) would demonstrate the existence of New Physics beyond the minimal extension of the SM necessary to induce the neutrino masses.

2.3 Radiative charged lepton decays $l \to l'\gamma$ in νSM

The MEG detector at the Paul Scherrer Institute (PSI) is searching for the lepton flavor violating decay $\mu^+ \to e^+\gamma$ since 2009. They recently have announced the new upper bound on the branching ratio of that decay, 5.7×10^{-13} at $90\% C.L.$ [9], using 3.6×10^{14} stopped muons, from data taken in 2009-2011. This bound is currently the strongest limit on this decay. The other bound is given by the experiment of MEGA, $Br(\mu \to e\gamma) \leq 1.2 \times 10^{-11}$ [10].

2.3.1 General expressions for $Br(l \to l'\gamma)$

As said before, if we consider masses of the neutrinos as an only extension of the SM, such model predicts an unobservably small branching ratio, $Br(\mu \to e\gamma) \leq 10^{-51}$. Thus, any observation of such decays would be an evidence of a New Physics (NP) beyond the SM.

To show the smallness of such decays in νSM and to analyze possible effects of NP, we first obtain the general form of decay rate of $l \to l'\gamma$.

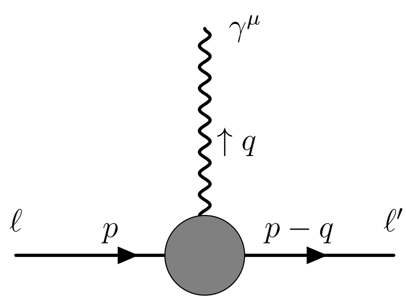


Figure 2.1: Momenta assignments for the $l \to l'\gamma$ decay.

The matrix element of the electromagnetic current operator between muon and electron can be written as (momenta assignments are shown in Fig. 2.1):

$$T_\lambda =< u_{l'}(p-q)|J_\lambda^{em}|u_l(p) >, \tag{2.19}$$

T_λ has the following Lorentz decomposition,

$$\begin{aligned} T_\lambda &= \bar{u}_{l'}(p-q)[\gamma^\mu(F_{VL}P_L + F_{VR}P_R) + (F_{SL}P_L + F_{SR}P_R)q^\mu \\ &+ i(F_{TL}\sigma^{\mu\nu}P_L + F_{TR}\sigma^{\mu\nu}P_R)q_\nu]u_l(p), \end{aligned} \tag{2.20}$$

where $F_{VL}, \dots F_{TR}$ are form factors multiplying various Dirac structures.

The electromagnetic gauge invariance requires for on-shell external particles:

$$\partial^\lambda J^{em}_\lambda = 0, \tag{2.21}$$

which can be translated into the following condition

$$- \quad m_e((F_{VL}P_L + F_{VR}P_R) + m_\mu(-F_{VL}P_L + F_{VR}P_R) + q^2(F_{SL}P_L + F_{SR}P_R) = 0, \tag{2.22}$$

leading finally to requirement $F_{VL} = F_{VR} = 0$ for $q^2 = 0$.

The amplitude of the decay reads:

$$M = \epsilon^\lambda(q) < u_{l'}(p-q)|J^{em}_\lambda|u_l(p) >, \tag{2.23}$$

where $\epsilon^\lambda(q)$ is the photon polarization vector. Since the vector form factors F_{VL}, F_{VR} vanish on-shell and $\epsilon^\lambda q_\lambda = 0$, the most general form of the on-shell amplitude (with the photon momentum $q^2 = 0$) is given by

$$M = \epsilon^\lambda \bar{u}_{l'}(p-q)[i(F_{TL}\sigma^{\mu\nu}P_L + F_{TR}\sigma^{\mu\nu}P_R)q_\nu]u_l(p). \tag{2.24}$$

A straightforward calculation of the decay rate yields

$$\Gamma(l \to l'\gamma) = \frac{m_l^3}{16\pi}(|F_{TL}|^2 + |F_{TR}|^2). \tag{2.25}$$

2.3.2 SM predictions for $Br(l \to l'\gamma)$

In the SM with massless neutrinos muon can decay only to electron and neutrino pair, $\mu^- \to e^- + \bar{\nu}_e + \nu_\mu$, other decays are forbidden kinematically or due to lepton flavor conservation. The decay width for such dominant standard channel can be used to normalize the branching ratios for more exotic LFV decays, so we remind it first.

The decay rate of muon decay, $\mu^-(p_\mu) \to e^-(p_e) + \bar{\nu}_e(k_{\nu_e}) + \nu_\mu(k_{\nu_\mu})$, where the momentum of each particle is given in the parenthesis, can be calculated as follows. Matrix element in the effective Fermi Lagrangian is given by

$$M = \frac{G_F}{\sqrt{2}}[\bar{u}(k_{\nu_\mu})\gamma_\rho(1-\gamma_5)u(p_\mu)][\bar{u}(p_e)\gamma^\rho(1-\gamma_5)v(k_{\nu_e})], \tag{2.26}$$

where to a very good approximation G_μ can be related to the electroweak coupling and the W mass as

$$G_F = \frac{\sqrt{2}}{8}\frac{g^2}{M_W^2}. \tag{2.27}$$

Summing over fermion polarizations and integrating over phase space one gets:

$$\Gamma_\mu = \frac{1}{\tau_\mu} = \frac{G_F^2 m_\mu^2}{4\pi^3}\int_0^{E_e^{max}} dE_e E_e^2 (1 - \frac{4}{3}\frac{E_e}{m_\mu}) = \frac{G_F^2 m_\mu^5}{192\pi^3}, \tag{2.28}$$

where the maximum of electron energy is $E_e^{max} = m_\mu/2$. This leads to muon lifetime $\tau_\mu \cong 2.2 \times 10^{-6}$ sec.

The width of leptonic decay of τ into electron or muon can be done just by replacing m_μ by m_τ. However, τ has also hadronic decay channels. Thus, its full lifetime is given by

$$\tau_\tau = \tau_\mu (\frac{m_\mu}{m_\tau})^5 \frac{Br(\tau^- \to e^- \bar{\nu}_e \nu_\tau)}{Br(\mu^- \to e^- \bar{\nu}_e \nu_\mu)}. \tag{2.29}$$

The current experimental data, $m_\tau = 1776.99$ MeV, $m_\mu = 105.658357$ MeV, $\tau_\mu = 2.19703 \times 10^{-6}$ sec, $\tau_\tau = 2.906 \times 10^{-13}$ sec, $Br(\tau^- \to e^- \bar{\nu}_e \nu_\tau) = 0.1784$ and $Br(\mu^- \to e^- \bar{\nu}_e \nu_\mu) = 100\%$, fulfill well this relation, confirming the universality of weak interactions, i.e. common value of G_F coupling for both μ and τ, with the 1‰ accuracy.

Radiative charged lepton decays $l \to l'\gamma$ can occur in the SM modified by the presence of neutrino mass, however they are very small. Appropriate calculation for the $\mu \to e\gamma$ decay can be find e.g. in [11] or in Cheng-Lee textbook [12], here we cite the most important results. The relevant diagrams are depicted in Fig. 2.2.

The direct but tedious calculation shows that, after neglecting the electron mass

$$\begin{aligned} F_{TL} &= 0 \\ F_{TR} &= e\frac{g^2}{4M_W^2}\frac{m_\mu}{32\pi^2}\delta_\nu, \end{aligned} \tag{2.30}$$

where δ_ν is the GIM suppression factor,

$$\delta_\nu = \sum_{i=1}^{3} U_{ei}^* U_{\mu i} (\frac{m_i^2}{M_W^2}), \tag{2.31}$$

using eq. (2.25) and $\Gamma(\mu \to e\bar{\nu}\nu) = G_F^2 m_\mu^5/192\pi^3$ one gets [13, 14].

$$BR(\mu \to e\gamma) = \frac{3\alpha}{32\pi}\delta_\nu^2 \cong (2.5 - 3.9) \times 10^{-55}, \tag{2.32}$$

where α is the fine structure constant.

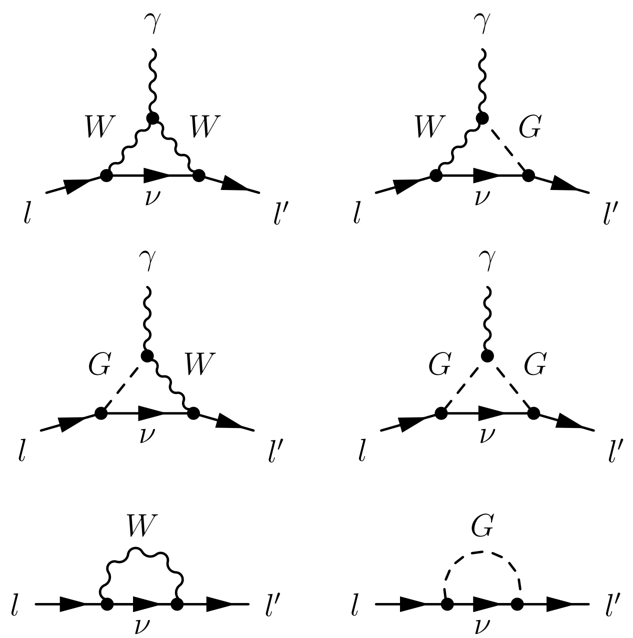


Figure 2.2: One-loop diagrams for the $l \to l'\gamma$ decay in the SM with massive neutrinos

2.4 $\ell_i \to \ell_j \ell_k \bar{\ell}_l$ decay in the SM

The calculation of branching ratio for the decay of $\mu \to eee$ in the νSM is more complicated as one needs to include more diagrams, apart from photon-penguin diagrams also the Z-penguins and box diagrams. The result is again extremely small, of the order of $\sim 10^{-50}$, unobservable in current or foreseeable experiment. Similar calculation can be used to estimate the rate $\tau \to \mu\mu\mu$ and $\tau \to eee$ decays, and as for muon they are unobservably small.

The current bound $BR(\mu \to eee) \sim 1 \times 10^{-12}$ has been set by SINDRUM experiment at PSI. The MU3e experiment has been proposed to improve the sensitivity by four orders of magnitude. The measured upper bounds for 3-body τ decays are set by various experiments. The LHC is a prolific source of τ leptons, the majority coming from D_s or B mesons. The limit of branching ratio for $\tau \to \mu\mu\mu$ at 7 TeV by LHCb collaboration [15] is 8×10^{-8} (90% C.L.) [16]. However, best upper limit at hadron collider is still worse than BELLE experiment: 2.1×10^{-8} (90% C.L.) [17].

The measurable rates for 3-body decays are predicted by various New Physics models. It is interesting to observe that the photon penguin contribution to the $l \to l'l'l'$ decays is enhanced by the logarithmic factor $\log(m_l^2/m_{l'}^2)$ and thus often dominant, at least in wide class of NP models. In such case the ratio of $Br(l \to l'\gamma)/Br(l \to 3l')$ is fixed and does not depend on any NP details. If such scenario is realized, some of the experiments measuring radiative and/or 3-body charged lepton decays may be redundant and not improve each other sensitivity range.

Chapter 3

Parametrization of the SM extensions in terms of effective operators

As already mentioned in Chapter 1, even if the Standard Model (SM) of strong and electroweak interactions has been successfully tested to a great precision [18], it is commonly accepted that it constitutes only an effective theory which is valid up an energy scale Λ where new physics (NP) enters and additional dynamic degrees of freedom become important. A renormalizable quantum field theory valid above this scale should satisfy the following requirements:

(*i*) Its gauge group must contain the SM gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$.

(*ii*) All SM degrees of freedom should be incorporated either as fundamental or as composite fields.

(*iii*) At low-energies it should reduce to the SM provided no undiscovered weakly coupled *light* particles exist (like axions or sterile neutrinos).

In most theories of physics beyond the SM that have been considered, the SM is recovered at low energies via the decoupling of the heavy particles with masses of the order of $\Lambda \gg M_Z$. That such a decoupling at the perturbative level is possible in a renormalization quantum field theory is guaranteed by the Appelquist-Carazzone decoupling theorem [19]. This leads to the appearance of higher-dimensional operators which are suppressed by powers of Λ and are added to the SM Lagrangian:

$$\mathcal{L}_{\rm SM} = \mathcal{L}_{\rm SM}^{(4)} + \frac{1}{\Lambda} \sum_k C_k^{(5)} Q_k^{(5)} + \frac{1}{\Lambda^2} \sum_k C_k^{(6)} Q_k^{(6)} + \mathcal{O}\left(\frac{1}{\Lambda^3}\right). \tag{3.1}$$

Here $\mathcal{L}_{\mathrm{SM}}^{(4)}$ is the usual renormalizable part of the SM Lagrangian which contains dimension-2 and dimension-4 operators only. $Q_k^{(5)}$ is the Weinberg operator giving rise to neutrino masses, $Q_k^{(6)}$ denote dimension-6 operators, and $C_k^{(n)}$ stand for the corresponding dimensionless coupling constants, i.e. the Wilson coefficients.

Even if the ultimate theory of NP at some high energy scale is not a quantum field theory, at low energies the effective theory still reduces to a quantum field theory [20] and it is possible to parametrize its effects at the electroweak scale in terms of these operators and the associated Wilson coefficients. Thus, one can search for NP in a model independent way by studying the SM extended with gauge invariant effective higher dimensional operators. Later, once a specific model is chosen, the Wilson coefficients can be calculated as a function of model parameters by matching the model of NP under consideration on the SM extended with such higher dimensional operators and one can calculate bounds on the specific model as well.

Flavor observables, especially flavor changing neutral current (FCNC) processes are an excellent probe of new physics since they are suppressed in the SM and therefore sensitive even to small NP contributions. This also means that these processes can stringently constrain the Wilson coefficients of the dimension-6 operators induced by NP. Especially the search for lepton flavor violation (LFV) is very promising since, as discussed in the previous Chapter, in the SM extended with massive neutrinos all flavor violating effects in the charged lepton sector are proportional to the very small neutrino masses and thus by far too small to be measurable in any foreseeable experiment. This in turn means that any observation of LFV would prove the existence of physics beyond the SM. In addition, LFV processes have the advantage of being "theoretically clean", i.e. they can be computed precisely without problems with non-perturbative QCD effects affecting similar observables in the quark sector.

Lepton flavor violating processes have been studied in great detail in many specific extensions of the SM. For example in the MSSM non-vanishing decay widths for LFV processes are generated by flavor non-diagonal SUSY breaking terms [21, 22, 23, 24, 25]. Also extending the MSSM with right-handed neutrinos by the seesaw mechanism [26] gives rise to LFV [27, 28, 29, 30, 31, 32, 33, 34, 35], as well as allowing for R-parity violation [36, 37, 38]. Other models like the littlest Higgs Model with T-Parity [39], two-Higgs-doublet models with generic flavor structures [40, 41, 42, 43] or models with an extended fermion sector [44] have sources of lepton flavor violation, too. In order to make models of New Physics consistent with the non-observation of LFV processes in Nature, the assumption of Minimal Flavor Violation [45] has been extended to the lepton sector (see e.g. [46, 47]). Flavor changing

τ decays have been studied in Ref. [48] in a model independent way taking into account a (reducible) set of four-lepton operators and the magnetic lepton operators. However, a detailed model independent analysis with all gauge invariant operators is still pending[1].

In the next Chapters we perform such a model independent analysis by considering the SM extended with all dimension-6 operators giving rise to lepton flavor violation which are invariant under the SM gauge group. We study the radiative lepton decays $\ell_i \to \ell_f \gamma$ and three-body charged lepton decays $\ell_i \to \ell_j \ell_k \ell_l$, as well as the anomalous magnetic moments and EDMs of charged leptons and the flavor violating $Z^0 \to \ell_i^- \ell_j^+$ decays.

It is worth noting that analyzing the LFV processes using the gauge-invariant basis of dimension-6 operators automatically assures that the final results are also gauge invariant and contain all relevant contributions. Otherwise, one risks including just subset of diagrams contributing to a given process. For example it is quite common in the literature to calculate in a model of NP only the effective flavor changing Z^0-boson coupling to charged leptons and neglect the corrections to W couplings, as the latter do not contribute at the tree-level to neutral current processes. However, both Z^0 and W (and also Goldstone boson) couplings come from the same set of gauge-invariant higher-order operators, and are thus of the same size. In fact, (as our calculation shows explicitly) their contributions at least to some processes, like e.g. $\ell_i \to \ell_f \gamma$, are equally important and should be always considered together.

The complete (but still reducible) list of independent operators of dimension-5 and dimension-6 which can be constructed out of SM fields and which are invariant under the SM gauge group fields was first derived in Ref. [52]. We follow the notation Ref. [8] where the operator basis of Ref. [52] was reduced to a minimal set. We list below again the operators relevant for our discussion. We use the following indices and symbols, based on notation introduced in Section 2.1:

- $a, b = 1, 2$ label the components of the weak isospin doublets.
- i, j, k, l are flavor indices running from 1 to 3.
- L and R stand for the chiralities.
- φ^a is the SM Higgs doublet where φ^2 is the neutral component.

[1]For a model independent analysis for the Higgs sector of the SM see Ref. [49, 50] and for anomalous top couplings Ref. [51, 49].

The hypercharges of the SM fields are summarized in Table 2.1. The sign conventions for the covariant derivatives are defined in eqs. (2.3,2.4). The hermitian derivative terms are ($\varphi^\dagger \overleftarrow{D}_\mu \varphi \equiv (D_\mu\varphi)^\dagger \varphi$):

$$\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi \equiv i\varphi^\dagger \left(D_\mu - \overleftarrow{D}_\mu \right) \varphi \quad \text{and} \quad \varphi^\dagger i \overleftrightarrow{D}^I_\mu \varphi \equiv i\varphi^\dagger \left(\tau^I D_\mu - \overleftarrow{D}_\mu \tau^I \right) \varphi \,. \tag{3.2}$$

In general, the SM can be extended by higher dimensional operators starting from dimension-5. However, there is just a single dimension-5 term respecting the SM gauge symmetry which, after electroweak symmetry breaking, generates neutrino masses and mixing angles - the Weinberg operator (C is the charge conjugation matrix and $\varepsilon_{12} = +1$):

$$Q_{\nu\nu} = \varepsilon_{ab}\varepsilon_{cd}\varphi^a\varphi^c(\ell^b_i)^T C \ell^d_j \,. \tag{3.3}$$

This operator does not contribute directly (other then modifying the U_{PMNS} matrix) to LFV processes in the charged lepton sector, consequently we do not consider it in the rest of our analysis.

In Table 3.1 we collect the independent dimension-6 operators relevant for our discussion, i.e. all operators which can contribute to LFV processes in the charged lepton sector at the tree-level or at the 1-loop level. We neglect the operators which could give LFV effects only via the interference with the dimension-4 SM vertices containing the PMNS matrix, since such effects are suppressed by the small neutrino masses which we assume to be zero. The names of operators in the left column of each block should be supplemented with generation indices of the fermion fields whenever necessary, e.g. $Q^{(1)}_{\ell q} \to Q^{(1)ijkl}_{\ell q}$. Dirac and color indices (not displayed) are always contracted within the brackets. The same is true for the isospin indices, except for $Q^{(1)}_{\ell equ}$ and $Q^{(3)}_{\ell equ}$.

Note that different flavor index combinations of the 4-lepton operators can correspond to the same operator (for example $Q^{ijkl}_{\ell\ell} = Q^{ilkj}_{\ell\ell} = Q^{kjil}_{\ell\ell} = Q^{klij}_{\ell\ell}$). For this reason, in the following we will only consider one of these combinations which avoids the introduction of combinatorial factors. This can be achieved by the requirement $i \geq k$, $j \geq l$ for $Q^{ijkl}_{\ell\ell,ee}$, so that the relevant part of the Lagrangian can be written as:

$$\mathcal{L} = \frac{1}{\Lambda^2} \sum_{ijkl, i\geq k, j\geq l} \left(C^{ijkl}_{\ell\ell} Q^{ijkl}_{\ell\ell} + C^{ijkl}_{ee} Q^{ijkl}_{ee} \right) + \frac{1}{\Lambda^2} \sum_{ijkl} C^{ijkl}_{\ell e} Q^{ijkl}_{\ell e} \,. \tag{3.4}$$

Note that for $C^{ijkl}_{\ell e}$ all possible flavor index permutations correspond to different operators. Due to the hermicity of the Lagrangian we find the additional relations like $C^{ijkl}_{\ell\ell} = C^{jilk\star}_{\ell\ell}$. Similar ones hold for all four-fermion operators.

$\ell\ell\ell\ell$		$\ell\ell X\varphi$		$\ell\ell\varphi^2 D$ and $\ell\ell\varphi^3$	
$Q_{\ell\ell}$	$(\bar{\ell}_i\gamma_\mu\ell_j)(\bar{\ell}_k\gamma^\mu\ell_l)$	Q_{eW}	$(\bar{\ell}_o\sigma^{\mu\nu}e_j)\tau^I\varphi W^I_{\mu\nu}$	$Q^{(1)}_{\varphi\ell}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi)(\bar{\ell}_i\gamma^\mu\ell_j)$
Q_{ee}	$(\bar{e}_i\gamma_\mu e_j)(\bar{e}_k\gamma^\mu e_l)$	Q_{eB}	$(\bar{\ell}_i\sigma^{\mu\nu}e_j)\varphi B_{\mu\nu}$	$Q^{(3)}_{\varphi\ell}$	$(\varphi^\dagger i\overleftrightarrow{D}^I_\mu\varphi)(\bar{\ell}_i\tau^I\gamma^\mu\ell_j)$
$Q_{\ell e}$	$(\bar{\ell}_i\gamma_\mu\ell_j)(\bar{e}_k\gamma^\mu e_l)$			$Q_{\varphi e}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi)(\bar{e}_i\gamma^\mu e_j)$
				$Q_{e\varphi 3}$	$(\varphi^\dagger\varphi)(\bar{\ell}_i e_j\varphi)$

$\ell\ell qq$					
$Q^{(1)}_{\ell q}$	$(\bar{\ell}_i\gamma_\mu\ell_j)(\bar{q}_k\gamma^\mu q_l)$	$Q_{\ell d}$	$(\bar{\ell}_i\gamma_\mu\ell_j)(\bar{d}_k\gamma^\mu d_l)$	$Q_{\ell u}$	$(\bar{\ell}_i\gamma_\mu l_j)(\bar{u}_k\gamma^\mu u_l)$
$Q^{(3)}_{\ell q}$	$(\bar{\ell}_i\gamma_\mu\tau^I\ell_j)(\bar{q}_k\gamma^\mu\tau^I q_l)$	Q_{ed}	$(\bar{e}_i\gamma_\mu e_j)(\bar{d}_k\gamma^\mu d_l)$	Q_{eu}	$(\bar{e}_i\gamma_\mu e_j)(\bar{u}_k\gamma^\mu u_l)$
Q_{eq}	$(\bar{e}_i\gamma^\mu e_j)(\bar{q}_k\gamma_\mu q_l)$	$Q_{\ell edq}$	$(\bar{\ell}^a_i e_j)(\bar{d}_k q^a_l)$	$Q^{(1)}_{\ell equ}$	$(\bar{\ell}^a_i e_j)\varepsilon_{ab}(\bar{q}^b_k u_l)$
				$Q^{(3)}_{\ell equ}$	$(\bar{\ell}^a_i\sigma_{\mu\nu}e_a)\varepsilon_{ab}(\bar{q}^b_k\sigma^{\mu\nu}u_l)$

Table 3.1: Complete list of the dimension-6 operators (invariant under the SM gauge group) which contribute to the LFV observables under consideration at the tree or at the one-loop level.

The dominant contributions to the processes considered in this thesis are given by diagrams with flavor changing gauge boson vertices or contact 4-fermion vertices. However, to preserve gauge-invariance, also Goldstone boson exchanges has to be taken into account even if, with few exceptions of mixed $W^\pm G^\mp$ diagrams, they are suppressed by additional powers of light lepton masses over v, the Higgs field VEV. In general, the operators listed in Table 3.1 give rise also to flavor violating physical Higgs boson couplings. We neglect them in our analysis as they are also of the higher order in m_ℓ/m_{h^0}.

The $(\varphi^\dagger\varphi)(\bar{\ell}_i e_j\varphi)$ operator does not contain gauge boson fields and modifies only Higgs and Goldstone boson couplings, which in principle could affect our results. However, it gives also new $\mathcal{O}(1/\Lambda^2)$ contribution to the charged lepton mass matrix:

$$m^\ell_{fi} = \frac{v}{\sqrt{2}}Y^\ell_f\delta_{fi} + \frac{v^3}{2\sqrt{2}\Lambda^2}C^{fi}_{e\varphi 3}\,. \tag{3.5}$$

The necessary rediagonalization of lepton masses has the effect of modifying the relation between the Yukawa coupling and the charged lepton masses (and the PMNS matrix). However, one can see that in the triple Goldstone boson couplings to leptons still the physical lepton masses and the physical PMNS matrix enter so the $Q^{fi}_{e\phi 3}$ does not generate flavor violation in these couplings. The triple coupling of the physical Higgs boson h^0 to charged leptons, as well as all quadruple and quintuple vertices derived from $Q_{e\phi 3}^{fi}$ can still be flavor violating. Nonetheless, their contributions to the processes discussed below vanish or are small due to an additional suppression of m_ℓ/m_{h^0}, compared to the dominant contributions from $Q_{\varphi e}$, $Q_{\varphi\ell}^{(1)}$ and $Q_{\varphi\ell}^{(3)}$ operators[2]. Thus, we neglect this operator (and thus the entire $\ell\ell\varphi^3$ class) in our analysis, provided that the rediagonalization of the lepton mass matrix has been performed.

The operators of the $\ell\ell X\varphi$ class (as defined in Table 3.1) can give rise to both radiative lepton decays and to three-body neutral current lepton decays already at the tree-level. The 4-lepton $\ell\ell\ell\ell$ operators and the operators of the $\ell\ell\varphi^2 D$ class can contribute to $\ell_i \to \ell_j\ell_k\ell_l$ decays at the tree-level and to $\ell_i \to \ell_f\gamma$ decays at the 1-loop level. Finally, the operators of the $\ell\ell qq$ class can contribute to both types of decays only at the 1-loop level. However, for 3-body decays we are only interested in the tree-level contributions and concerning the radiative lepton decays, it turns out that only $Q_{\ell equ}^{(3)}$ gives a non-zero contribution.

In the Appendix A we list the Feynman rules arising from the operators given in Table 3.1 which are necessary in order to calculate the flavor observables discussed in the next Chapters.

[2] $O_{e\varphi 3}^{fi}$ generates flavour-changing couplings of the SM Higgs. The resulting effects have been studied in Refs. [53, 54, 55]

Chapter 4

$\ell_J \to \ell_I\gamma$ decay rate

In this chapter we study the radiative lepton decays $\mu \to e\gamma$, $\tau \to \mu\gamma$ and $\tau \to e\gamma$, for which we calculate both the tree level and the one loop predictions, taking into account all dimension-6 operators.

Process	Experimental bounds
$\mathcal{B}[\tau \to \mu\gamma]$	$\leq 4.4 \times 10^{-8}$ [56, 57]
$\mathcal{B}[\tau \to e\gamma]$	$\leq 3.3 \times 10^{-8}$ [56]
$\mathcal{B}[\mu \to e\gamma]$	$\leq 5.7 \times 10^{-13}$ [58]

Table 4.1: Experimental upper limits on the branching ratios of the radiative lepton decays.

In Table 4.1 we listed the current experimental bounds on the radiative lepton decays $\ell_J \to \ell_I\gamma$. Up to now no signal is found. The strongest bound exist for $\mu \to e\gamma$ decay and will be further improved in the MEG experiment at PSI, where the planned sensitivity of searches for this decay should reach 10^{-16}. Bounds for $\tau \to \mu(e)\gamma$ decays are weaker, and simultaneously can have the largest probability in many models describing LFV. This has motivated many experimental searches [57].

The $\ell_J \to \ell_I\gamma$ decay have been studied in many specific extension of the SM. Here we present a model independent analysis, expressing the branching ratios in terms of Wilson coefficients of the operators of dimension-5 and -6, i.e. up to the order of $\frac{1}{\Lambda^2}$.

Adding explicit flavor indices in eq. (2.20), the general form of lepton-photon vertex can be written as:

$$\begin{aligned} V^{IJ\,\mu}_{\ell\ell\gamma} &= \frac{i}{\Lambda^2}[\gamma^\mu(F^{IJ}_{VL}P_L + F^{IJ}_{VR}P_R) + (F^{IJ}_{SL}P_L + F^{IJ}_{SR}P_R)q^\mu \\ &+ i(F^{IJ}_{TL}\sigma^{\mu\nu}P_L + F^{IJ}_{TR}\sigma^{\mu\nu}P_R)q_\nu]. \end{aligned} \tag{4.1}$$

Gauge invariance requires that F_{VL} and F_{VR} must vanish for on-shell external particles. The form factors F_{SL} and F_{SR} does not contribute to the $\ell_J \to \ell_I\gamma$ decay amplitudes. Thus, only the form factors F_{TL} and F_{TR} contribute to $\ell^J \to \ell^I\gamma$ decay and the general form of the branching ratio can be expressed in terms of F^{IJ}_{TL} and F^{IJ}_{TR} as follows:

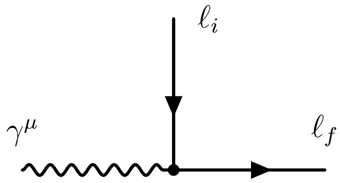
$$\mathcal{B}\left(\ell^J \to \ell^I\gamma\right) = \frac{m^3_{\ell_J}}{16\pi\Lambda^4\,\Gamma_{\ell_J}}\left(\left|F^{IJ}_{TR}\right|^2 + \left|F^{IJ}_{TL}\right|^2\right)\,, \tag{4.2}$$

where Γ_{ℓ_J} is the total decay width of decaying lepton, $\Gamma_\mu = \frac{G_F^2 m_\mu^5}{192\pi^3}$ and Γ_τ also includes the hadronic channels.

4.1 Branching ratio for the $\ell_J \to \ell_I\gamma$ at tree level

Only the operators Q_{eW} and Q_{eB} can give contribution to $V^{IJ\,\mu}_{\ell\ell\gamma}$ at the three level. If their coefficients are comparable to other Wilson coefficients of the dimension 6 operators, they dominate the effective photon-lepton vertex, with the form factors simply given by:

$$F^{IJ}_{TR} = F^{IJ\star}_{TL} = v\sqrt{2}\left(c_W C^{IJ}_{eB} - s_W C^{IJ}_{eW}\right). \tag{4.3}$$



$$i(\gamma^\mu\delta_{fi} + \tfrac{iv\sqrt{2}}{\Lambda^2}\sigma^{\mu\nu}\left[C^{fi}_{\gamma L}P_L + C^{fi}_{\gamma R}P_R\right]q_\nu)$$

$$C^{fi}_{\gamma R} = C^{fi*}_{\gamma L} = C_W C^{fi}_{eB} - S_W C^{fi}_{eW}$$

Figure 4.1: Effective lepton-photon vertex

The branching ratio for the decay of the $\ell_J \to \ell_I\gamma$ at tree level can be written as:

$$\mathcal{B}\left(\ell^J \to \ell^I\gamma\right) = \frac{m^3_{\ell_J}v^2}{4\pi\Lambda^4\,\Gamma_{\ell_J}}\left|c_W C^{IJ}_{eB} - s_W C^{IJ}_{eW}\right|^2\,, \tag{4.4}$$

This analytical results can be compared (normalized) to the experimental bounds on radiative lepton charged decays given in Table 4.1. If they are dominated by tree-level contributions from Q_{eB} and Q_{eW}, one gets

$$\begin{aligned}
\sqrt{\left|C_\gamma^{12}\right|^2+\left|C_\gamma^{21}\right|^2} &\leq 2.45\times 10^{-10}\left(\frac{\Lambda}{1\ \mathrm{TeV}}\right)^2\sqrt{\frac{\mathrm{Br}\left[\mu\to e\gamma\right]}{5.7\times 10^{-13}}}\,, \\
\sqrt{\left|C_\gamma^{13}\right|^2+\left|C_\gamma^{31}\right|^2} &\leq 2.35\times 10^{-6}\left(\frac{\Lambda}{1\ \mathrm{TeV}}\right)^2\sqrt{\frac{\mathrm{Br}\left[\tau\to e\gamma\right]}{3.3\times 10^{-8}}}\,, \\
\sqrt{\left|C_\gamma^{23}\right|^2+\left|C_\gamma^{32}\right|^2} &\leq 2.71\times 10^{-6}\left(\frac{\Lambda}{1\ \mathrm{TeV}}\right)^2\sqrt{\frac{\mathrm{Br}\left[\tau\to \mu\gamma\right]}{4.4\times 10^{-8}}}\,.
\end{aligned} \tag{4.5}$$

If the New Physics scale in low, in the TeV range, the bounds on the coefficients are very strict, for the decay $\mu\to e\gamma$ of the order of 10^{-10} and for decays $\tau\to\mu\gamma, e\gamma$ of the order of 10^{-6}.

4.2 Branching ratio for the $\ell_J\to\ell_I\gamma$ at one loop level

In the renormalizable theories beyond the SM, the operators Q_{eB} and Q_{eW} can only be generated at the loop level while other operators, like the effective four lepton couplings, can already be generated at tree level. In some theories of the extended SM, C_{eW} and C_{eB} may not even generated at all [59]. Thus, the radiative lepton decays contributions can come from other dimension-6 operators which contribute to such decays at 1-loop level. These contributions can be comparable or even dominant with tree level contributions. The generic topologies of the diagrams at one loop level in the order of $1/\Lambda^2$ and the relevant momenta assignments are shown in Fig. 4.2.

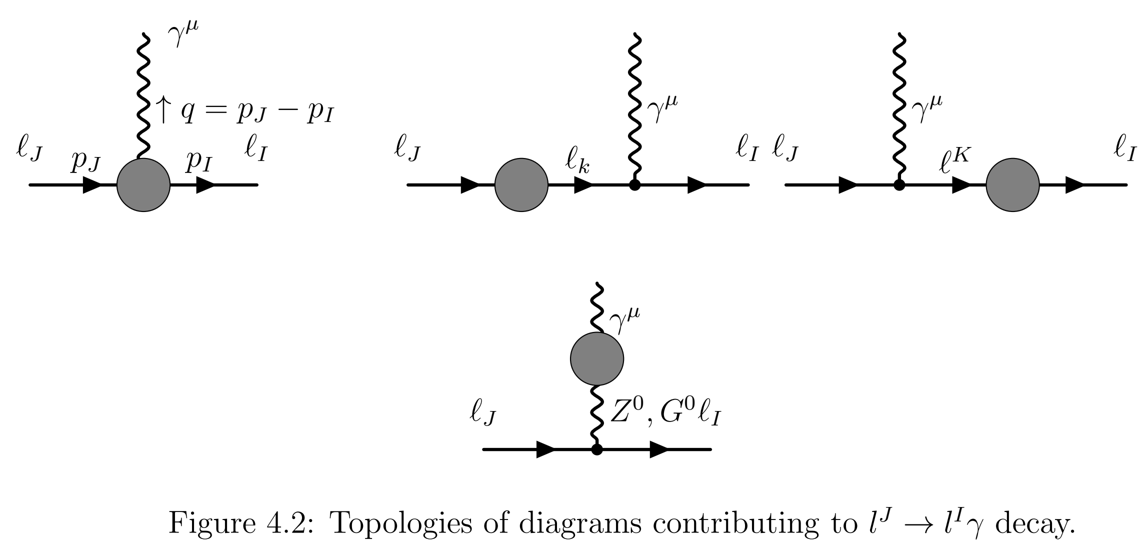


Figure 4.2: Topologies of diagrams contributing to $l^J\to l^I\gamma$ decay.

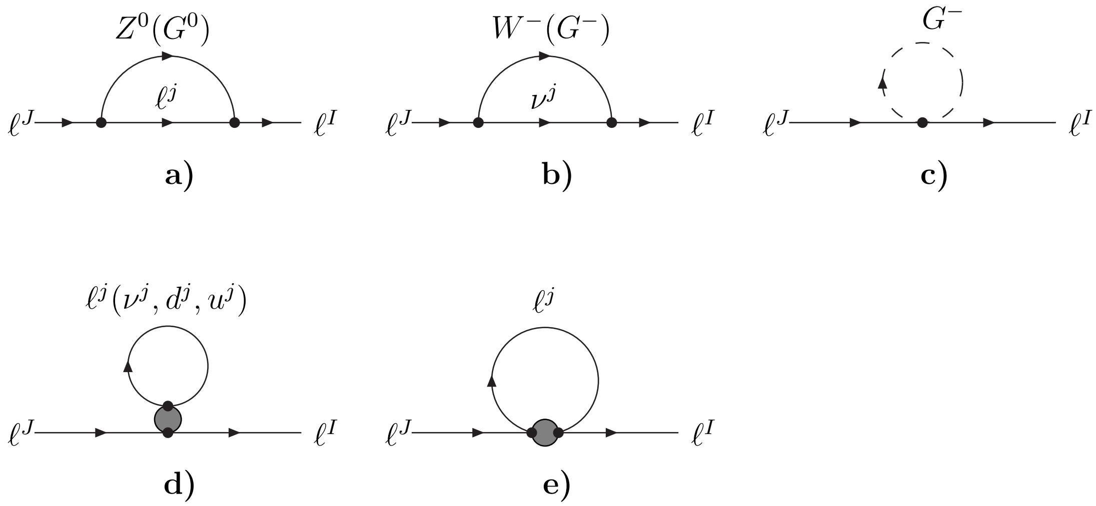


Figure 4.3: Diagrams contributing to LFV self-energy of charged leptons.

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The list of all one loop diagrams contributing to the effective $l^I \to l^J\gamma$ amplitude is given in Fig. 4.3 (self energy diagrams) and in Fig. 4.4 (1-particle irreducible diagrams). In our loop calculation we do not consider lepton-lepton-Z and lepton-lepton-photon flavor violating generated by the operators Q_{eW} and Q_{eB} because if their coefficients are not negligible, their contribution would dominate the decay already at the tree level.

Since the diagrams involved dimension-6 vertices, the loop integrals have complicated tensor structure. Thus, evaluation of many diagrams shown in Figs. 4.3 and 4.4 is rather tedious and technically non-trivial. Performing such calculations by hand would be very time consuming and very prone to misprints. Thus, an extensive set of Mathematica routines has been developed, using the Tracer package [60] for Dirac matrices manipulations. In addition, to have an additional test of correctness of the final results, calculations has been repeated using two different methods. In first of them, full Passarino-Veltman decomposition of loop functions has been done, followed by their expansions in the small mass ratios. In the second method, the amplitudes of diagrams has been expanded in momenta of external particles (so-called "Asymptotic Expansion") before performing the integration and later simpler version of loop integrals, valid for vanishing external momenta, has been used. Both approaches lead to the same result. In this thesis we discuss in more details the first (more universal) method, using the Passarino-Veltman decomposition.

In this approach technically we first calculated all loop diagrams exactly using the R_ξ gauge with $\xi = 1$ (in the second method using the Asymptotic Expansion $\xi = 1$ was not

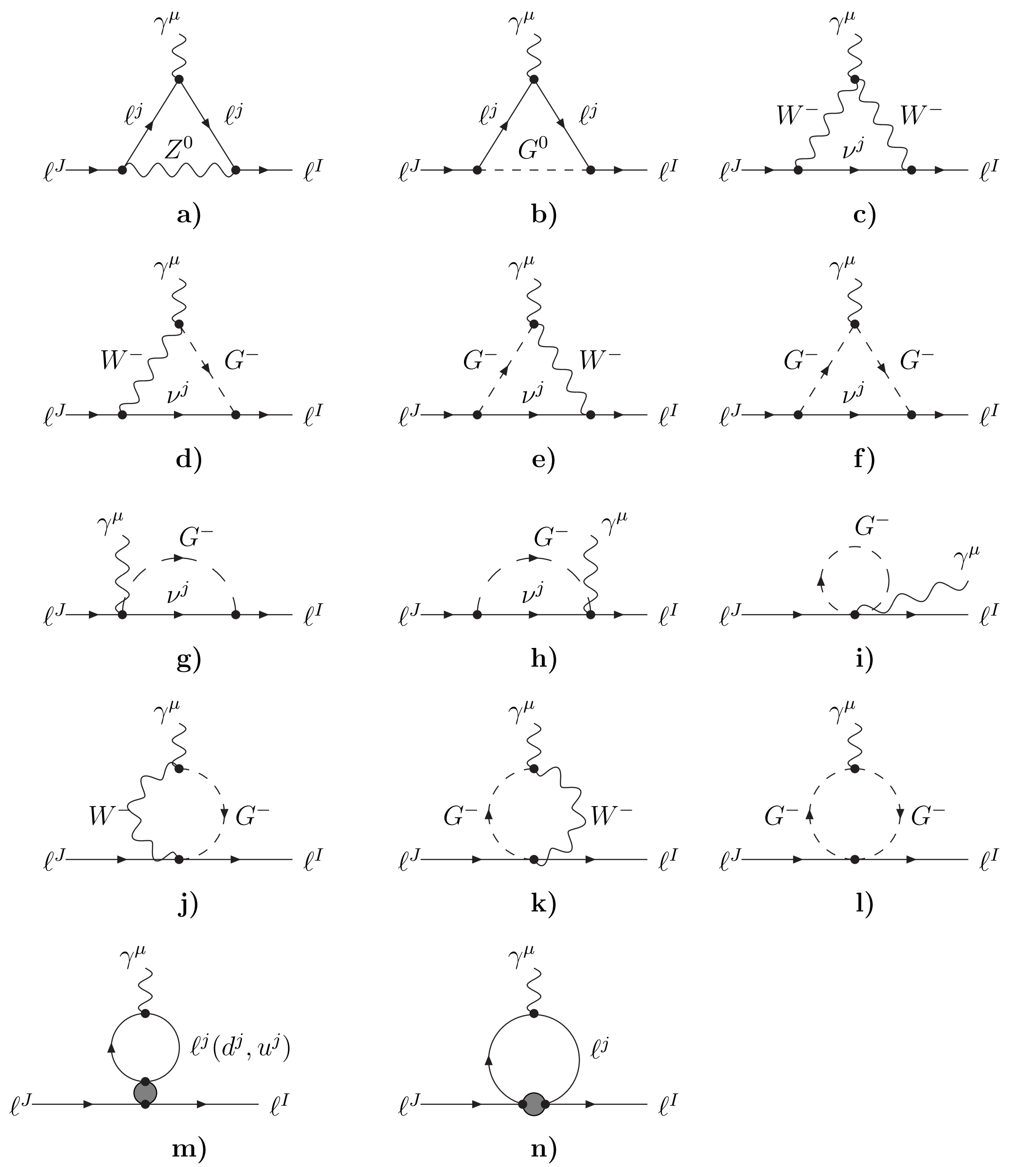

Figure 4.4: 3-point 1PI diagrams contributing to the radiative charged lepton decay $\ell^J \to \ell^I \gamma$ at the 1-loop level.

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assumed!) and expressing all amplitudes in terms of Passarino-Veltman a_i, b_{ij}, c_ij loop functions, as defined in Appendix B. At this stage we kept only the leading terms of the order of $1/\Lambda^2$, neglecting higher $1/\Lambda$ powers. Later we expanded all diagrams involving Z, W and Goldstone bosons in the charged lepton masses, retaining only the leading terms, m_ℓ/m_W and m_ℓ/m_Z. The powers of lepton mass were coming both from the on-shell external lepton momenta as well as the mass of leptons circulating in loops, and both possibilities have been accounted for. In principle all calculations has been automatized with the use of Mathematica code, however for illustration in the Appendix 6 as an example we have included the detailed calculation for the Z boson contribution to the one particle irreducible and self energy diagrams.

As mentioned above, the gauge invariance requires that vector form factors of flavor violating lepton-photon vertex vanish in a case that external particle are on-shell. We have used the vanishing of F_{VL} and F_{VR} as an additional check of our calculation. For that, we considered the self energy diagrams (both of lepton and photon), even if the 1PI diagrams are sufficient for the calculating the tensor form factors.

In the final result only 5 Wilson coefficients contribute to tensor form factors. Our final result for contributions from various types of exchanged particles to the form factors F_{TL} and F_{TR} are given in Table 4.2.

Operator $Q^{(3)}_{\ell equ}$ generates the divergent contribution, which can be understood and renormalized in a complete theory of New Physics. As such term contains quark fields and can be constrained from hadronic decays, we do not consider it in our numerical analysis. By using the LHC measurements, the coefficients from the Q^3_{lequ} operator, lepton quark contact term, could be constrained independently [61].

The summed finite 1-loop results for F_{TL}, F_{TR} form-factors can be written down as:

$$
\begin{aligned}
F^{IJ}_{TL} &= \frac{4e}{(4\pi)^2}\left[\frac{C^{(1)IJ}_{\phi l} m_I(1+s^2_W) - (C^{(3)IJ}_{\phi l} m_I + C^{IJ}_{\phi e} m_J)(\frac{3}{2}-s^2_W)}{3} + \sum_{K=1}^{3} C^{IKKJ}_{\ell e} m_K\right], \\
F^{IJ}_{TR} &= \frac{4e}{(4\pi)^2}\left[\frac{C^{(1)IJ}_{\phi l} m_J(1+s^2_W) - (C^{(3)IJ}_{\phi l} m_J + C^{IJ}_{\phi e} m_I)(\frac{3}{2}-s^2_W)}{3} + \sum_{K=1}^{3} C^{KJIK}_{\ell e} m_K\right].
\end{aligned}
$$

As an example of application of 1-loop expression for $\mathcal{B}\left(\ell^J \to \ell^I \gamma\right)$, we discuss bounds on the Wilson coefficient C_{le} of the contact 4-lepton coupling Q_{le}. It is interesting because for some choices of the flavor indices such operator does not contribute directly (at the tree-level) to the 3-body charged lepton decays discussed in Chapter 5. For our example we consider $\mu \to e\gamma$ decay and assume that at the tree-level it is dominated by C^{12}_{eW} term and

Particle in the loop	Tensor form-factors
Z^0	$F_{TL}^{Z\,fi} = \dfrac{4e\left[\left(C_{\varphi\ell}^{(1)fi} + C_{\varphi\ell}^{(3)fi}\right) m_f(1+s_W^2) - C_{\varphi e}^{fi} m_i(\frac{3}{2} - s_W^2)\right]}{3(4\pi)^2}$ $F_{TR}^{Z\,fi} = \dfrac{4e\left[\left(C_{\varphi\ell}^{(1)fi} + C_{\varphi l}^{(3)fi}\right) m_i(1+s_W^2) - C_{\varphi e}^{fi} m_f(\frac{3}{2} - s_W^2)\right]}{3(4\pi)^2}$
W	$F_{TL}^{W\,fi} = -\dfrac{10em_f C_{\varphi\ell}^{(3)fi}}{3(4\pi)^2}$ $F_{TR}^{W\,fi} = -\dfrac{10em_i C_{\varphi\ell}^{(3)fi}}{3(4\pi)^2}$
$G^0, G^\pm, WG$	$F_{TL}^{WG\,fi} = 0$ $F_{TR}^{WG\,fi} = 0$
qq	$F_{TL}^{4f\,fi} = -\dfrac{16e}{3(4\pi)^2} \sum_{j=1}^3 C_{\ell equ}^{(3)fijj\star} m_{u_j} \left(\Delta - \log\dfrac{m_{u_j}^2}{\mu^2}\right)$ $F_{TR}^{4f\,fi} = -\dfrac{16e}{3(4\pi)^2} \sum_{j=1}^3 C_{\ell equ}^{(3)fijj} m_{u_j} \left(\Delta - \log\dfrac{m_{u_j}^2}{\mu^2}\right)$
ll	$F_{TL}^{4\ell\,fi} = \dfrac{2e}{(4\pi)^2} \sum_{j=1}^3 C_{\ell e}^{fjji} m_j$ $F_{TR}^{4\ell\,fi} = \dfrac{2e}{(4\pi)^2} \sum_{j=1}^3 C_{\ell e}^{jifj} m_j$

Table 4.2: 1-loop contributions to form factors F_{TL}^{fi} and F_{TL}^{fi} giving rise to $\ell_i \to \ell_f \gamma$ up to order $1/\Lambda^2$.

other contributions vanish (so $C_{eB}^{12} \approx C_{\phi l}^{(1)12} \approx C_{\phi l}^{(3)12} \approx C_{\phi e}^{12} \approx 0$). Thus, we get the following expression for F_{TL} and F_{TR} form-factors:

$$F_{TL} = \frac{2e}{(4\pi)^2} \sum_{j=1}^{3} C_{\ell e}^{1jj2} m_j - v\sqrt{2} s_W C_{eW}^{12},$$
$$F_{TR} = \frac{2e}{(4\pi)^2} \sum_{j=1}^{3} C_{\ell e}^{j21j} m_j - v\sqrt{2} s_W C_{eW}^{12}. \tag{4.6}$$

We also neglect the coefficient proportional to masses of electron and muon ($k = 1, 2$), i.e. $C_{\ell e}^{1112}, C_{\ell e}^{1222}, C_{\ell e}^{1211}$ and $C_{\ell e}^{2212}$, leaving only the term proportional to the mass of taon

$(k = 3)$, obtaining the following expression for the branching ratio:

$$
\begin{aligned}
Br(\mu \to e\gamma) &= \frac{192\pi^3}{16\pi G_F^2 m_\mu^2 \Lambda^4}\left[\left|\frac{2em_\tau}{(4\pi)^2}C_{\ell e}^{1332} - v\sqrt{2}s_W C_{eW}^{12}\right|^2\right. \\
&+ \left.\left|\frac{2em_\tau}{(4\pi)^2}C_{\ell e}^{3213} - v\sqrt{2}s_W C_{eW}^{12}\right|^2\right]. \qquad (4.7)
\end{aligned}
$$

In Fig.4.5 the allowed regions in the $C_{\ell e}^{1332}$–$C_{\ell e}^{3213}$ plane for $\Lambda = 1$ TeV are shown. As can be seen, bounds on both coefficients are of the order of 10^{-5}, but the central value for C_{le}^{3213} shifts depending on C_{eW}^{12}.

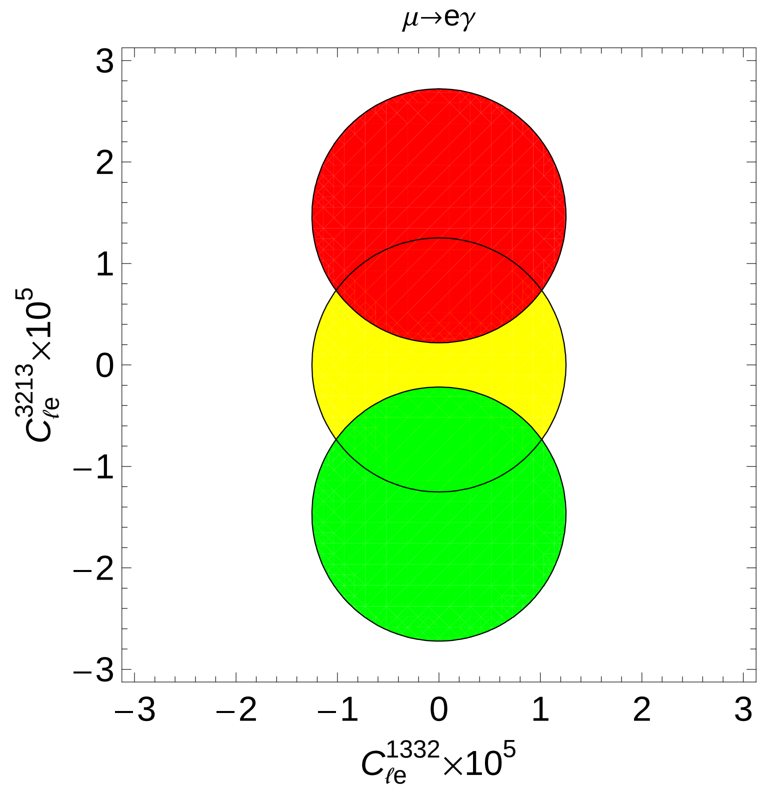

Figure 4.5: Allowed regions from Br$[\mu \to e\gamma]$ in the $C_{\ell e}^{1332}$–$C_{\ell e}^{3213}$ plane for $\Lambda = 1$ TeV and different values of C_{eW}^{12}. Yellow: $C_{eW}^{12} = 0$, red: $C_{eW}^{12} = 6 \times 10^{-8}$, green: $C_{eW}^{12} = -6 \times 10^{-8}$.

4.3 Vector and scalar form-factors contributing to off-shell $\ell_i \to \ell_f \gamma^*$ amplitude.

As mentioned above, gauge invariance requires that F_{VL} and F_{VR} ("vector") form-factors vanish for the on-shell external particles. Thus, expressions for them must be proportional to the momentum of the outgoing photon and they do not contribute to $\ell_i \to \ell_f \gamma$ decay rate. The "scalar" form-factors F_{SL} and F_{SR} does not need to vanish on-shell, but they also cancel out from this amplitude after contracting with the photon polarization vector. Still, those form-factors can enter the expressions for the more complicated processes. Thus, for completeness we list them below, again split into groups of contributions within which the vector form-factors vanish in the on-shell limit. Note that some of them are infinite and require renormalization.

We give only expressions for left scalar form-factor F_{SL} - the right one can be obtained from F_{SL} by changing the sign and exchanging the external fermion masses, i.e.:

$$F_{SR} = -F_{SL}(m_i \leftrightarrow m_f) \tag{4.8}$$

Z^0 loops:

$$F_{VL}^{Z\,fi} = \frac{2e(1-2s_W^2)Q^2}{9(4\pi)^2}\left(C_{\varphi\ell}^{(1)fi} + C_{\varphi l}^{(3)fi}\right)\left(1 - 6\log\frac{m_i m_f}{M_Z^2}\right), \tag{4.9}$$

$$F_{VR}^{Z\,fi} = -\frac{4es_W^2 Q^2}{9(4\pi)^2} C_{\varphi e}^{fi}\left(1 - 6\log\frac{m_i m_f}{M_Z^2}\right), \tag{4.10}$$

$$F_{SL}^{Z\,fi} = \frac{2e}{9(4\pi)^2}\left[m_f(1-2s_W^2)\left(C_{\varphi\ell}^{(1)fi} + C_{\varphi l}^{(3)fi}\right) + 2m_i s_W^2 C_{\varphi e}^{fi}\right]\left(1 - 6\log\frac{m_i m_f}{M_Z^2}\right). \tag{4.11}$$

WG loops:

$$\begin{aligned}F_{VL}^{WG\,fi} &= -\frac{2eQ^2}{9(4\pi)^2}\left[16C_{\varphi l}^{(3)fi} + 6c_W^2\left(C_{\varphi\ell}^{(1)fi} + C_{\varphi l}^{(3)fi}\right)\right.\\ &\quad\left. + 3c_W^2\left(15C_{\varphi\ell}^{(1)fi} + 16C_{\varphi l}^{(3)fi}\right)\left(\Delta - \log\frac{M_W^2}{\mu^2}\right)\right],\end{aligned} \tag{4.12}$$

$$F_{VR}^{WG\,fi} = -\frac{2ec_W^2 Q^2}{3(4\pi)^2} C_{\varphi e}^{fi}\left[2 + 15\left(\Delta - \log\frac{M_W^2}{\mu^2}\right)\right], \tag{4.13}$$

$$\begin{aligned}F_{SL}^{WG\,fi} &= \frac{e}{9(4\pi)^2}\left[12c_W^2(m_i C_{\varphi e}^{fi} - m_f(C_{\varphi\ell}^{(1)fi} + C_{\varphi l}^{(3)fi})) - 32m_f C_{\varphi l}^{(3)fi}\right.\\ &\quad\left. + 3\left(15c_W^2(m_i C_{\varphi e}^{fi} - m_f(C_{\varphi\ell}^{(1)fi} + C_{\varphi l}^{(3)fi})) - 2m_f C_{\varphi l}^{(3)fi}\right)\left(\Delta - \log\frac{M_W^2}{\mu^2}\right)\right].\end{aligned} \tag{4.14}$$

G^0 loops:

$$F_{VL}^{G^0 fi} = F_{VR}^{G^0 fi} = F_{SL}^{G^0 fi} = F_{SR}^{G^0 fi} = 0 \tag{4.15}$$

$G^\pm$ loops:

$$F_{VL}^{G^\pm fi} = F_{VR}^{G^\pm fi} = F_{SL}^{G^\pm fi} = F_{SR}^{G^\pm fi} = 0 \tag{4.16}$$

$4l$ loops - contact 4-lepton diagrams:

$$F_{VL}^{4\ell\, fi} = -\frac{2eQ^2}{3(4\pi)^2}\sum_{j=1}^{3}\left(2C_{\ell\ell}^{fijj} + C_{\ell e}^{fijj}\left(\Delta - \log\frac{m_{\ell_j}^2}{\mu^2}\right)\right), \tag{4.17}$$

$$F_{VR}^{4\ell\, fi} = -\frac{2eQ^2}{3(4\pi)^2}\sum_{j=1}^{3}\left(2C_{ee}^{fijj} + C_{\ell e}^{jjfi}\left(\Delta - \log\frac{m_{\ell_j}^2}{\mu^2}\right)\right), \tag{4.18}$$

$$F_{SL}^{4\ell\, fi} = -\frac{2e}{3(4\pi)^2}\sum_{j=1}^{3}\left(2C_{\ell\ell}^{fijj}m_f - 2C_{ee}^{fijj}m_i - (C_{\ell e}^{jjfi}m_i - C_{\ell e}^{fijj}m_f)\left(\Delta - \log\frac{m_{\ell_j}^2}{\mu^2}\right)\right). \tag{4.19}$$

$4f$ loops - contact 2-lepton-2-quark diagrams:

$$\begin{aligned} F_{VL}^{4f\, fi} &= \frac{4eQ^2}{9(4\pi)^2}\sum_{j=1}^{3}\left(C_{\ell q}^{(1)fijj} - C_{\ell q}^{(3)fijj} + C_{\ell u}^{fijj}\right)\left(\Delta - \log\frac{m_{u_j}^2}{\mu^2}\right) \\ &- \frac{2eQ^2}{9(4\pi)^2}\sum_{j=1}^{3}\left(C_{\ell q}^{(1)fijj} + C_{\ell q}^{(3)fijj} + C_{\ell d}^{fijj}\right)\left(\Delta - \log\frac{m_{d_j}^2}{\mu^2}\right), \end{aligned} \tag{4.20}$$

$$\begin{aligned} F_{VR}^{4f\, fi} &= \frac{4eQ^2}{9(4\pi)^2}\sum_{j=1}^{3}\left(C_{eq}^{fijj} + C_{eu}^{(3)fijj}\right)\left(\Delta - \log\frac{m_{u_j}^2}{\mu^2}\right) \\ &- \frac{2eQ^2}{9(4\pi)^2}\sum_{j=1}^{3}\left(C_{eq}^{fijj} + C_{ed}^{(3)fijj}\right)\left(\Delta - \log\frac{m_{d_j}^2}{\mu^2}\right), \end{aligned} \tag{4.21}$$

$$\begin{aligned} F_{SL}^{4f\, fi} &= \frac{4e}{9(4\pi)^2}\sum_{j=1}^{3}\left(m_f\left(C_{\ell q}^{(1)fijj} - C_{\ell q}^{(3)fijj} + C_{\ell u}^{fijj}\right)\right. \\ &- \left. m_i\left(C_{eq}^{fijj} + C_{eu}^{(3)fijj}\right)\right)\left(\Delta - \log\frac{m_{u_j}^2}{\mu^2}\right) \\ &- \frac{2e}{9(4\pi)^2}\sum_{j=1}^{3}\left(m_f\left(C_{\ell q}^{(1)fijj} + C_{\ell q}^{(3)fijj} + C_{\ell d}^{fijj}\right)\right. \\ &- \left. m_i\left(C_{eq}^{fijj} + C_{ed}^{(3)fijj}\right)\right)\left(\Delta - \log\frac{m_{d_j}^2}{\mu^2}\right). \end{aligned} \tag{4.22}$$

4.4 Electric and magnetic moments of charged leptons

In this Section we investigate the possible size of the anomalous magnetic and electric dipole moments of charged leptons in the extension of the SM with operators of dimension-6.

The most general Lorentz-invariant form of the coupling of a photon-lepton with q_ν the 4 momentum of photon is given by [62]:

$$\Gamma^\mu = F_1(q^2)\gamma^\mu + F_2(q^2)\frac{i}{2m_\ell}\sigma^{\mu\nu}q_\nu - F_3(q^2)\sigma^{\mu\nu}\gamma^5 q_\nu. \tag{4.1}$$

Where values of form factors for $q^2 = 0$ give:

- the electric charge of the lepton $Q_\ell = F_1(q^2 = 0)$.
- the anomalous magnetic moment of the lepton $a_{\ell i} = \frac{(g_{\ell i}-2)}{2} = F_2(q^2 = 0)$.
- the electric dipole moment of the lepton $\frac{d_\ell}{Q_\ell} = F_3(q^2 = 0)$.

All of F_i has to be real because of Hermicity of the SM Lagrangian.

4.5 Anomalous magnetic moments

Magnetic moment is a fundamental quantity, related to the particle "gyromagnetic ratio" g, and given by:

$$\vec{\mu} = g(\frac{q}{2m})\vec{S}, \tag{4.2}$$

where $q = \pm|e|$ is charge of the particle with spin $\frac{1}{2}$ and $g = 2$ for the structureless particle. The anomalous magnetic moment is defined by:

$$a_\ell = \frac{1}{2}(g_\ell - 2). \tag{4.3}$$

The anomalous magnetic moment (AMM) provides an excellent test for physics beyond the SM. The anomalous magnetic moment of electron is very precisely measured and calculated in the SM (where the largest uncertainty comes from the measurement of the fine structure constant α_{em}, $\alpha_{em}^{-1} = 137.03599884(91)$ from a Rubidium atom experiment [63]).

To achieve the accuracy of theoretical prediction for $q-2$ matching the experimental precision, one needs to calculate a very large number of Feynman diagrams, up to 5-loop level for $g_e - 2$ [64], obtaining

$$g^{theory}/2 - 1 = 0.0011596521818(8); \tag{4.4}$$

in agreement with experimental measurement [65]:

$$g^{experiment}/2 - 1 = 0.0011596521807(3). \tag{4.5}$$

Even if the anomalous magnetic of electron is most precisely measured, the anomalous magnetic moment of muon is more sensitive to potential LFV in the New Physics models. LFV effects could be even stronger for the τ lepton AMM, however the current experimental limits on it is rather weak (but it can be improved in the future [66]):

$$-0.052 \leq a_\tau \leq 0.013\,. \tag{4.6}$$

In the SM anomalous magnetic moment of muon receives three dominant contributions (Fig. 4.6):

$$a_\mu^{SM} = a_\mu^{QED} + a_\mu^{EW} + a_\mu^{Had}, \tag{4.7}$$

where a_μ^{QED} includes all photonic and leptonic loop contributions [67, 68, 64, 69, 70, 71], a_μ^{EW} the loop contributions involving the $W^\pm, Z$ and Higgs bosons [72, 73] (both known very precisely) and a_μ^{Had} contains contributions from hadronic loops, dominating the theoretical uncertainly. The calculation of these contributions and relevant analysis can be found in [72, 74]. The SM result is one of the very few of its predictions differing significantly from the experimentally measured value:

$$\delta a_\mu = a_\mu^{exp} - a_\mu^{SM} = 287(63)(49) \times 10^{-11}. \tag{4.8}$$

The new experiments aiming to measure the muon anomalous magnetic moment more accurately will start in Fermilab in the $g-2$ BNL E821 experiment [75] around 2016.

Many models of New Physics could explain this discrepancy (the leading candidates are supersymmetric theories [76, 77, 78, 79, 80, 81, 82, 83]) In our model-independent approach the formulae for charged lepton AMM can be extracted from the effective lepton-photon vertex defined in eq. (4.1) and read as:

$$a_{\ell_i} = \frac{2m_{\ell_i}}{e\Lambda^2}\,\mathrm{Re}\left[F_{TR}^{ii}\right]\,. \tag{4.9}$$

Using the results of Table 4.2, one can obtain simplified numerical expressions for the anomalous magnetic moments of charged leptons. Neglecting small lepton mass ratios and taking into account that some of the Wilson coefficients of the 4-lepton and the Z-lepton

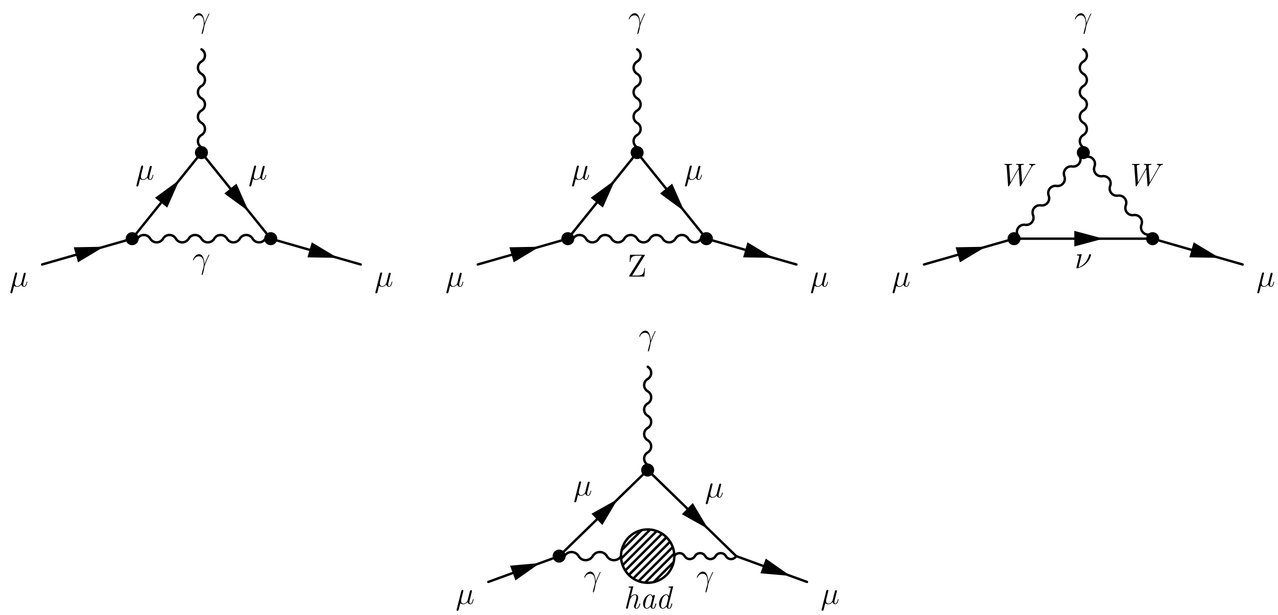

Figure 4.6: Diagrams contributing to a_μ^{SM}. From left to right, first order of QED, lowest order of electroweak, lowest order of hadronic.

vertices are real in the flavor conserving case, we find the following expressions:

$$\begin{aligned} a_e &= 1.17 \times 10^{-6} \;\mathrm{Re}\left[2 \times 10^{-5}\, C_{\ell e}^{3113} + C_\gamma^{11}\right] \left(\frac{1\ \mathrm{TeV}}{\Lambda}\right)^2 , \\ a_\mu &= 2.43 \times 10^{-4} \;\mathrm{Re}\left[2 \times 10^{-5}\, C_{\ell e}^{3223} + C_\gamma^{22}\right] \left(\frac{1\ \mathrm{TeV}}{\Lambda}\right)^2 , \\ a_\tau &= 4.1 \times 10^{-3} \;\mathrm{Re}\Big[10^{-5} \times \Big(1.6\, C_{\varphi\ell}^{(1)33} + 2.0\, C_{\ell e}^{3333} \\ &- 1.7 \left(C_{\varphi\ell}^{(3)33} + C_{\varphi e}^{33}\right)\Big) + C_\gamma^{33}\Big] \left(\frac{1\ \mathrm{TeV}}{\Lambda}\right)^2 . \end{aligned} \tag{4.10}$$

Comparing the expression above to experimental measurements, one can easily read off the bounds on Wilson coefficients contributing to lepton AMMs.

4.6 Electric Dipole Moments

A permanent Electric Dipole Moment (EDM) of a fundamental particle violates both time-reversal T and space-parity P symmetries. Assuming CPT symmetry, measuring EDM is another way to measure also CP violation (with C denoting the charge conjugation C). CP violation is one of the Sakharov's three conditions to generate the matter anti-matter asymmetry from the initially symmetrical phase. Test of these discrete symmetries have been very important in establishing the structure of SM.

The non-relativistic Hamiltonian of a particle of spin S can be written as:

$$H = -\mu B.\frac{S}{s} - dE.\frac{S}{s}, \tag{4.11}$$

where s is the value of particle spin. Under the reflection of spacial coordinates, $P(B.S) = B.S$ and $P(E.S) = -E.S$. Thus, a nonzero value for d is a hint for existence of parity violation. d also break time-reversal invariance, as under time reflection, $T(B.S) = B.S$ and $T(E.S) = -E.S$. Therefore, measurements showing that both parity and time-reversal are not conserved suggest a nonzero value of electric dipole moments. In quantum field theory, electric dipole moment corresponds to the following term in the Lagrangian:

$$\mathcal{L}_{EDM} = -i\frac{d_f}{2}\bar{f}_{L/R}\sigma^{\mu\nu}\gamma_5 f_{L/R}F_{\mu\nu}, \tag{4.12}$$

(as usual L and R are chirality indices). The current experimental bounds of charged lepton EDMs are listed in Table 4.3. New EDM measurements planned in near future aim to improve the sensitivity by two orders of magnitude, testing the New Physics models to higher mass scales in the 20-100 TeV range.

EDMs	$\lvert d_e \rvert$	$\lvert d_\mu \rvert$	d_τ
Bounds (e cm)	8.7×10^{-29} [84]	1.9×10^{-19} [85]	$\in [-2.5,\, 0.8] \times 10^{-17}$ [86]

Table 4.3: Experimental upper bounds on electric dipole moments of the charged leptons.

In the SM the only CP violating parameter is the phase of the CKM matrix. It generates quark EDMs, but they are very small, not sufficient to explain matter-antimatter asymmetry in the Universe. Thus, we need to additional CP violating phases in physics beyond the SM.

Again, general expression for EDM can be obtained from effective lepton-photon interaction of eq. (4.1):

$$d_{\ell_i} = \frac{-1}{\Lambda^2}\,\mathrm{Im}\left[F_{TR}^{ii}\right]\,, \tag{4.13}$$

It can be used to find numerical expressions for the bounds on Wilson coefficients resulting from the EDMs, similar to those obtained from AMM measurements :

$$\begin{aligned}
d_e &= -2.08 \times 10^{-18}\ \mathrm{Im}\left[2 \times 10^{-5}\, C_{\ell e}^{3113} + C_{\gamma}^{11}\right]\left(\frac{1\ \mathrm{TeV}}{\Lambda}\right)^2 e\,\mathrm{cm}\,,\\
d_\mu &= -2.08 \times 10^{-18}\ \mathrm{Im}\left[2 \times 10^{-5}\, C_{\ell e}^{3223} + C_{\gamma}^{22}\right]\left(\frac{1\ \mathrm{TeV}}{\Lambda}\right)^2 e\,\mathrm{cm}\,,\\
d_\tau &= -2.08 \times 10^{-18}\ \mathrm{Im}\left[C_{\gamma}^{33}\right]\left(\frac{1\ \mathrm{TeV}}{\Lambda}\right)^2 e\,\mathrm{cm}\,.
\end{aligned} \tag{4.14}$$

Chapter 5

3-body charged lepton decays $\ell_i \to \ell_j \ell_k \bar{\ell}_l$

In this Chapter we investigate the three body decays of a heavy lepton to three lighter leptons in the context of the SM extended with operators of dimension-6. In this case such decays can be generated at tree level directly by 4-lepton operators or by photon, Z boson or neutral Goldstone boson exchanges, and we only consider such tree level contributions.

3-body Lepton Flavour Violating decays of the taon, $\tau^- \to \mu^-\mu^+\mu^-, \tau^- \to e^-e^+e^-$ and of muon, $\mu^- \to e^-e^+e^-$ are very strongly suppressed and completely negligible in the SM. Thus, as for radiative lepton decays, their observation would be a clear evidence for physics beyond the SM, possibly at scales far from the reach of direct observation in the LHC.

Belle and BaBar experiments searched for the following τ^- decays into three leptons: $e^-e^+e^-$, $\mu^-\mu^+\mu^-$, $e^-\mu^+\mu^-$, $\mu^-e^+e^-$, $\mu^-e^+\mu^-$ and $e^-\mu^+e^-$, reaching 90% CL upper limits on the branching fractions of the order of 10^{-8}[17]. For the decay of $\mu^+ \to e^+e^-e^+$ the currently best upper limit obtained by SINDRUM experiment is 1.0×10^{-12}[87]. Mu3e experiment at PSI is also going to search for the decay $\mu^+ \to e^+e^-e^+$. This experiment is planned in two phases. In the first phase (2014-2017), it should reach a sensitivity of BR($\mu^+ \to e^+e^-e^+$)$\approx 10^{-15}$. In the second phase, the experiment will reach the sensitivity of BR($\mu^+ \to e^+e^-e^+$)$\approx 10^{-16}$. The experimental bounds on all measured 3-body lepton decays are summarized in Table 5.1.

It is worth noting that at muon experiments, a proton beam collides on a target producing charged pions. Pions slow down interacting with the target matter and decay at rest into a muon and neutrino. If we neglect the small correction due to the mass of the neutrino,

Process	Experimental bound
$\mathcal{B}\left[\tau^- \to \mu^-\mu^+\mu^-\right]$	2.1×10^{-8} [17]
$\mathcal{B}\left[\tau^- \to e^-e^+e^-\right]$	2.7×10^{-8} [17]
$\mathcal{B}\left[\tau^- \to e^-\mu^+\mu^-\right]$	2.7×10^{-8} [17]
$\mathcal{B}\left[\tau^- \to \mu^-e^+\mu^-\right]$	1.7×10^{-8} [17]
$\mathcal{B}\left[\mu^- \to e^-e^+e^-\right]$	1.0×10^{-12} [88]

Table 5.1: Experimental upper limits on the branching ratios of the three body charged lepton decays.

muons are 100% polarized. The muons are transmitted to the another target and also stop and decay at rest. Based on the setup of the experiment, muons of either positive or negative charge can be selected to be transmitted to the second target. Negative muons would form bound states with atoms in the second target. Thus, the experimental bounds usually is given on the positive muon decays at free states.

$\ell_i \to \ell_j \ell_k \bar{\ell}_l$ decay rate

In this Section we obtain the formulae for the branching ratios for all the possible three body lepton decays, taking into account the tree level contributions from the LFV operators of dimension-6.

In Figure 5.1 we show the kinematics of the $\ell_i \to \ell_j \ell_k \bar{\ell}_l$ decay in the center of mass frame.

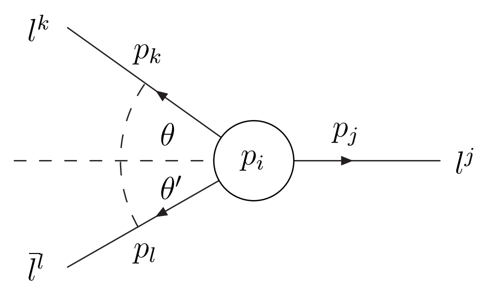

Figure 5.1: Kinematics of $\ell_i \to \ell_j \ell_k \bar{\ell}_l$ decay in the CMS frame.

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The expression for the 3-body decay branching ratio depends on the type of particles in the final state. One can distinguish 3 possibilities:

(A) Three lepton with the same flavor in the final state: $\mu^\pm \to e^\pm e^+ e^-$, $\tau^\pm \to e^\pm e^+ e^-$ and $\tau^\pm \to \mu^\pm \mu^+ \mu^-$.

(B) Three different (distinguishable) leptons in the final state: $\tau^\pm \to e^\pm \mu^+ \mu^-$ and $\tau^\pm \to \mu^\pm e^+ e^-$.

(C) Two lepton of the same flavor (opposite charge) in the final state: $\tau^\pm \to e^\mp \mu^\pm \mu^\pm$ and $\tau^\pm \to \mu^\mp e^\pm e^\pm$.

To calculate the decay rate we decomposed the amplitude into two parts, denoted A_γ for the sum of diagrams with photon exchange (generated by the operators Q_{eW} and Q_{eB}) and A_0 for sum of all other diagrams (contact 4-lepton interactions, Z- and neutral Goldstone-boson exchanges).

$$A = A_0 + A_\gamma. \tag{5.1}$$

In A_0 one can neglect the momentum dependence in the heavy boson propagators. Then, using if necessary also appropriate Fierz transformations, this amplitude in general can be written down as follows:

$$A_0 = \frac{1}{\Lambda^2} \sum_I C_I [\bar{u}(p_j) Q_I u(p_i)][\bar{u}(p_k) Q'_I v(p_l)], \tag{5.2}$$

where Q_I and Q'_I are various Dirac structures listed below and C_I can be expressed in terms of Wilson coefficients of dimension-6 operators. In general we can write the basis of quadrilinears $Q_I \times Q'_I$ as:

$$\begin{aligned} O_{VXY} &= \gamma^\mu P_X \times \gamma^\mu P_Y, \\ O_{SXY} &= P_X \times P_Y, \\ O_{TX} &= \sigma^{\mu\nu} \times \sigma^{\mu\nu} P_X, \end{aligned} \tag{5.3}$$

where X, Y stands for the chiralities L and R.

The contributions from photon exchanges for different types of decays (A), (B) and (C) are:

$$\begin{aligned} A_\gamma^{(A)} &= \frac{ev}{\Lambda^2} \Bigg(\frac{1}{(p_i - p_j)^2} [\bar{u}(p_j) i\sigma^{\mu\nu} (C_{\gamma L} P_L + C_{\gamma R} P_R)(p_i - p_j)_\nu u(p_i)][\bar{u}(p_k) \gamma_\mu v(p_l)] \\ &- (p_j \leftrightarrow p_k) \Bigg), \\ A_\gamma^{(B)} &= \frac{ev}{\Lambda^2} \frac{1}{(p_i - p_j)^2} [\bar{u}(p_j) i\sigma^{\mu\nu} (C_{\gamma L} P_L + C_{\gamma R} P_R)(p_i - p_j)_\nu u(p_i)][\bar{u}(p_k) \gamma_\mu v(p_l)], \\ A_\gamma^{(C)} &= 0. \end{aligned} \tag{5.4}$$

In the calculation of the spin averaged square matrix element $\mathcal{M} = \frac{1}{2}\sum_{pol}|A|^2$, in the $|A_0|^2$ one can neglected the masses of the lighter leptons, assuming $m_i \equiv M \gg m_j, m_k, m_l$. As discussed in more details below, light lepton masses in the photon penguin contribution must be treated with more care and can be neglected only after phase space integration. After all simplifications, the general form of the spin averaged square matrix element is:

$$\mathcal{M} = \mathcal{M}_0 + \mathcal{M}_\gamma, \tag{5.5}$$

where

$$\begin{aligned}
\mathcal{M}_0 &= \frac{8}{\Lambda^4}\left[(p_jp_k)(p_jp_l + p_kp_l)(|C_{VLL}|^2 + |C_{VRR}|^2)\right.\\
&+ (p_jp_l)(p_jp_k + p_kp_l)(|C_{VLR}|^2 + |C_{VRL}|^2)\\
&+ \frac{1}{4}(p_jp_k + p_jp_l)(p_kp_l)(|C_{SLL}|^2 + |C_{SRR}|^2 + |C_{SLR}|^2 + |C_{SRL}|^2)\\
&+ 4[(p_jp_l)(p_kp_l) + (p_jp_k)(p_kp_l) + 4(p_jp_k)(p_jp_l)](|C_{TL}|^2 + |C_{TR}|^2)\\
&+ \left. 2(p_jp_l - p_jp_k)(p_kp_l)\mathrm{Re}(C_{SLL}C^\star_{TL} + C_{SRR}C^\star_{TR})\right],
\end{aligned} \tag{5.6}$$

and the photon contribution depends again on the process type:

$$\begin{aligned}
\mathcal{M}_\gamma^{(A)} &= -\frac{8evM}{\Lambda^4}\mathrm{Re}\left[p_jp_l(C_{VLR}C^\star_{\gamma R} + C_{VRL}C^\star_{\gamma L}) + 2p_jp_k(C_{VLL}C^\star_{\gamma R}\right.\\
&+ \left. C_{VRR}C^\star_{\gamma L}) - \frac{1}{2}p_kp_l(C_{SLR}C^\star_{\gamma R} + C_{SRL}C^\star_{\gamma L})\right] + \frac{4e^2v^2}{\Lambda^4}\left[5p_jp_k - M^2\right.\\
&+ \left.\frac{M^4 + (4p_jp_k - M^2)^2}{4}\left(\frac{1}{(p_k+p_l)^2} + \frac{1}{(p_j+p_l)^2}\right)\right](|C_{\gamma L}|^2 + |C_{\gamma R}|^2),\\
\mathcal{M}_\gamma^{(B)} &= -\frac{8evM}{\Lambda^4}\mathrm{Re}\left[p_jp_l(C_{VLR}C^\star_{\gamma R} + C_{VRL}C^\star_{\gamma L}) + p_jp_k(C_{VLL}C^\star_{\gamma R} + C_{VRR}C^\star_{\gamma L})\right]\\
&+ \frac{2e^2v^2}{\Lambda^4}\left[4p_jp_k - M^2 + \frac{M^4 + (4p_jp_k - M^2)^2}{2(p_k+p_l)^2}\right](|C_{\gamma L}|^2 + |C_{\gamma R}|^2)\\
\mathcal{M}_\gamma^{(C)} &= 0.
\end{aligned} \tag{5.7}$$

Using standard expressions for massless 3-particle phase space one can integrate the $\mathcal{M}_0$ part matrix element and obtain the branching ratio of the form:

$$\begin{aligned}
\mathrm{Br}(\ell_i \to \ell_j\ell_k\bar{\ell}_l) &= \frac{N_cM^5}{6144\pi^3\Lambda^4\Gamma_{\ell_i}}\left(4\left(|C_{VLL}|^2 + |C_{VRR}|^2 + |C_{VLR}|^2 + |C_{VRL}|^2\right)\right.\\
&+ |C_{SLL}|^2 + |C_{SRR}|^2 + |C_{SLR}|^2 + |C_{SRL}|^2\\
&+ \left. 48\left(|C_{TL}|^2 + |C_{TR}|^2\right) + X_\gamma\right),
\end{aligned} \tag{5.8}$$

where $N_c = 1/2$ if two of the final state leptons are identical and $N_c = 1$ in all other cases, X_γ denotes the photon contribution and Γ^{ℓ_i} is the total decay width of the initial lepton.

The phase space integral for the photon penguin contribution (and its interference with other terms) X_γ is much more difficult and delicate, as photon propagator is singular in the limit of vanishing external lepton masses. In this case, to get the correct result, one needs to perform spin-average of the matrix element keeping light lepton mass terms up to m_i^2/M^2, later start from the full mass-dependent expressions for the 3-body final state kinematics, expand them also at least to m_i^2/M^2, do the phase space integration and only in the final result neglect all subleading terms in m_i/M. Result of such procedure (known earlier in the literature and derived independently here) gives the logarithmic enhancement factor $\log\frac{M^2}{m^2}$ for the pure photon penguin contribution (M, m are the masses of heaviest and next-to heaviest lepton in given decay process):

$$
\begin{aligned}
X_\gamma^{(A)} &= -\frac{16ev}{M}\mathrm{Re}\left[\left(2C_{VLL} + +C_{VLR} - \frac{1}{2}C_{SLR}\right)C^\star_{\gamma R}\right. \\
&+ \left.\left(2C_{VRR} + C_{VRL} - \frac{1}{2}C_{SRL}\right)C^\star_{\gamma L}\right] \\
&+ \frac{64e^2v^2}{M^2}(\log\frac{M^2}{m^2} - \frac{11}{4})(|C_{\gamma L}|^2 + |C_{\gamma R}|^2),
\end{aligned}
$$

$$
\begin{aligned}
X_\gamma^{(B)} &= -\frac{16ev}{M}\mathrm{Re}\left[\left(C_{VLL} + C_{VLR}\right)C^\star_{\gamma R} + \left(C_{VRR} + C_{VRL}\right)C^\star_{\gamma L}\right] \\
&+ \frac{32e^2v^2}{M^2}(\log\frac{M^2}{m^2} - 3)(|C_{\gamma L}|^2 + |C_{\gamma R}|^2) \\
X_\gamma^{(C)} &= 0.
\end{aligned} \tag{5.9}
$$

The quantities C_X in eqs. (5.8,5.9) can be expressed in terms of Wilson coefficients of operators in Table 3.1. Their specific form depends on the final state composition (decays (A), (B) and (C)) and we list them in the following sections.

5.1 Decays of group A: $\ell_i \to \ell_j\ell_j\bar{\ell}_j$

This option responds to the physical decays $\mu \to 3e$, $\tau \to 3e$ and $\tau \to 3\mu$. In general, at the tree-level diagrams mediated by Z^0, the neutral Goldstone boson and 4-lepton diagram from contact terms can contribute to the matrix element. The quantities C_X read as:

$$
\begin{aligned}
C_{VLL} &= 2\left((2s_W^2-1)\left(C_{\phi\ell}^{(1)ji}+C_{\phi\ell}^{(3)ji}\right)+C_{\ell\ell}^{jijj}\right), \\
C_{VRR} &= 2\left(2s_W^2 C_{\phi e}^{ji}+C_{ee}^{jijj}\right), \\
C_{VLR} &= -\frac{1}{2}C_{SRL}=\left(2s_W^2\left(C_{\phi\ell}^{(1)ji}+C_{\phi\ell}^{(3)ji}\right)+C_{le}^{jijj}\right), \\
C_{VRL} &= -\frac{1}{2}C_{SLR}=\left((2s_W^2-1)C_{\phi e}^{ji}+C_{\ell e}^{jjji}\right), \\
C_{SLL} &= C_{SRR}=C_{TL}=C_{TR}=0, \\
C_{\gamma L} &= 2\sqrt{2}s_W\left(c_W C_{eB}^{if\star}-s_W C_{eW}^{if\star}\right), \\
C_{\gamma R} &= 2\sqrt{2}s_W\left(c_W C_{eB}^{fi}-s_W C_{eW}^{fi}\right).
\end{aligned}
\tag{5.10}
$$

5.2 Decays of group B: $\ell_i \to \ell_j \ell_k \bar{\ell}_k$

Such a decay can be realized by $\tau^\pm \to e^\pm \mu^+ \mu^- e$ or $\tau^\pm \to \mu^\pm e^+ e^-$. The coefficients C_X read

$$
\begin{aligned}
C_{VLL} &= \left((2s_W^2-1)\left(C_{\phi\ell}^{(1)ji}+C_{\phi l}^{(3)ji}\right)+C_{ll}^{jikk}\right), \\
C_{VRR} &= \left(2s_W^2 C_{\phi e}^{ji}+C_{ee}^{jikk}\right), \\
C_{VLR} &= \left(2s_W^2\left(C_{\phi l}^{(1)ji}+C_{\phi\ell}^{(3)ji}\right)+C_{\ell e}^{jikk}\right), \\
C_{VRL} &= \left((2s_W^2-1)C_{\phi e}^{ji}+C_{\ell e}^{jkki}\right), \\
C_{SLR} &= -2C_{\ell e}^{jkki}, \\
C_{SRL} &= -2C_{\ell e}^{jikk}, \\
C_{SLL} &= C_{SRR}=C_{TL}=C_{TR}=0, \\
C_{\gamma L} &= 2\sqrt{2}s_W\left(c_W C_{eB}^{if\star}-s_W C_{eW}^{if\star}\right), \\
C_{\gamma R} &= 2\sqrt{2}s_W\left(c_W C_{eB}^{fi}-s_W C_{eW}^{fi}\right).
\end{aligned}
\tag{5.11}
$$

5.3 Decays of group C: $\ell_i^\pm \to \bar{\ell}_j^\mp \ell_k^\pm \ell_k^\pm$

As in previous Section, only τ lepton can decay into such channels, $\tau^\pm \to e^\mp \mu^\pm \mu^\pm e$ or $\tau^\pm \to \mu^\mp e^\mp e^\mp$. In this case Z-mediated diagrams are suppressed by $1/\Lambda^4$ and only contact 4$-$lepton diagram can contribute to these (rather exotic) process. The coefficients C_X are given by:

$$
\begin{aligned}
C_{VLL} &= 2C_{\ell\ell}^{kikj}, \\
C_{VRR} &= 2C_{ee}^{kikj},
\end{aligned}
$$

$$
\begin{aligned}
C_{VLR} &= -\frac{1}{2}C_{SRL} = C_{le}^{kikj}, \\
C_{VRL} &= -\frac{1}{2}C_{SLR} = C_{\ell e}^{kjki}, \\
C_{SLL} &= C_{SRR} = C_{TL} = C_{TR} = 0, \\
C_{\gamma L,R} &= 0.
\end{aligned} \tag{5.12}
$$

5.4 Numerical Applications

Analyzing the expressions for the radiative and for the 3-body lepton decays, one can see that (using current experimental bounds) the coefficient C_γ^{fi} defining the strength of the non-standard photon-lepton coupling is more strongly constrained by the former. Thus, one can neglect contribution from C_γ^{fi} to 3-body decays, and use them to derive formulae constraining Wilson coefficients of other operators. Neglecting small lepton masses, here we find:

$$
\begin{aligned}
C_{\mu eee} &\leq 3.29 \times 10^{-5}\left(\frac{\Lambda}{1\ \text{TeV}}\right)^2 \sqrt{\frac{\text{Br}\left[\mu \to eee\right]}{1 \times 10^{-12}}}, \\
C_{\tau eee} &\leq 1.28 \times 10^{-2}\left(\frac{\Lambda}{1\ \text{TeV}}\right)^2 \sqrt{\frac{\text{Br}\left[\tau \to eee\right]}{2.7 \times 10^{-8}}}, \\
C_{\tau \mu\mu\mu} &\leq 1.13 \times 10^{-2}\left(\frac{\Lambda}{1\ \text{TeV}}\right)^2 \sqrt{\frac{\text{Br}\left[\tau \to \mu\mu\mu\right]}{2.1 \times 10^{-8}}},
\end{aligned} \tag{5.13}
$$

with $C_{\ell_i\ell_f\ell_f\ell_f}$ given by

$$
\begin{aligned}
C_{\ell_i\ell_f\ell_f\ell_f} = \Bigg\{ & \left|0.46\left(\mathrm{C}_{\phi\ell}^{(1)fi} + \mathrm{C}_{\phi\ell}^{(3)fi}\right) + \mathrm{C}_{\ell e}^{fiff}\right|^2 + 2\left|\mathrm{C}_{\ell e}^{fiff} - 0.54\left(\mathrm{C}_{\phi\ell}^{(1)fi} + \mathrm{C}_{\phi\ell}^{(3)fi}\right)\right|^2 \\
& + \left|\mathrm{C}_{\ell e}^{fffi} - 0.54\mathrm{C}_{\phi e}^{fi}\right|^2 + 2\left|\mathrm{C}_{ee}^{fiff} + 0.46\mathrm{C}_{\phi e}^{fi}\right|^2 \Bigg\}.
\end{aligned} \tag{5.14}
$$

Many dimension-6 operators contribute both to the expressions for radiative lepton decays and for the three-body charged lepton decays. Thus, their decay rates can be correlated. As known in the literature and discussed already in Section 2.4, the ratio of both decay rates in case in which only C_γ^{fi} is non-zero depends solely on SM parameters and is given by $1/(\frac{\alpha}{3\pi}(\log\frac{m_i^2}{m_f^2}-\frac{11}{4}))$. It is interesting (particularly for projecting new experiments which would measure each of the decays separately) how strongly this ratio could be modified by contributions other then photon penguin.

Such ratio is independent of the scale Λ of NP and depend only on the ratios of Wilson coefficients. Thus, given a specific model, one can determine the branching ratio for one process in terms of the other one independently of the scale of New Physics.

As an example, let's assume that only the Wilson coefficients C_γ^{ij}, $C_{\varphi e}^{ji}$ and $C_{\varphi\ell}^{(1)ji}$ do not vanish. For such scenario the relevant expressions read as:

$$
\begin{aligned}
Br(\ell_i\to\ell_j\ell_j\bar{\ell}_j) &= \frac{1}{64G_F^2\Lambda^4\Gamma_{\ell_i}}[(16(2s_W^2-1)^2+8s_W^4)\mid C_{\varphi\ell}^{(1)ij}\mid^2+(16s_W^4-5(2s_W^2-1)^2)\mid C_{\varphi e}^{ij}\mid^2 \\
&- \frac{16ev}{m_i}Re[\left(14s_W^2-4\right)C_{\varphi\ell}^{(1)ij}C_\gamma+\left(12s_W^2-2\right)C_{\varphi e}^{ij}C_\gamma] \\
&+ \frac{128e^2v^2}{m_i^2}(\log\frac{m_i^2}{m_j^2}-\frac{11}{4})\mid C_\gamma\mid^2] \qquad (5.15) \\
Br(\ell_i\to\ell_j\gamma) &= \frac{m_i^3}{16\pi\Lambda^4\Gamma_{\ell_i}}[4v^2\mid C_\gamma\mid^2+\frac{16e^2(1+s_W^2)^2}{9(4\pi)^4}m_i^2\mid C_{\varphi\ell}^{(1)ij}\mid^2+\frac{16e^2(\frac{3}{2}-s_W^2)^2}{9(4\pi)^2}m_i^2\mid C_{\varphi e}^{ij}\mid^2 \\
&+ \frac{8ev\sqrt{2}(1+s_W^2)}{3(4\pi)^2}m_iC_\gamma C_{\varphi\ell}^{(1)ij}+\frac{8ev\sqrt{2}(\frac{3}{2}-s_W^2)^2}{3(4\pi)^2}m_iC_\gamma C_{\varphi e}^{ij}]. \qquad (5.16)
\end{aligned}
$$

In Fig. 5.2 we show the ratios $\mathrm{Br}\left[\ell_i\to\ell_f\gamma\right]/\mathrm{Br}\left[\ell_i\to\ell_f\ell_f\bar{\ell}_f\right]$ as a function of $\frac{C_{\varphi e}^{fi}}{C_\gamma^{fi}}$ and $\frac{C_{\varphi\ell}^{(1)\,fi}}{C_\gamma^{fi}}$. The value predicted in the photon-penguin domination scenario responds to point $(0,0)$ in each plot. As one can see, contributions from other operators can change this ratio even by factor of 2.

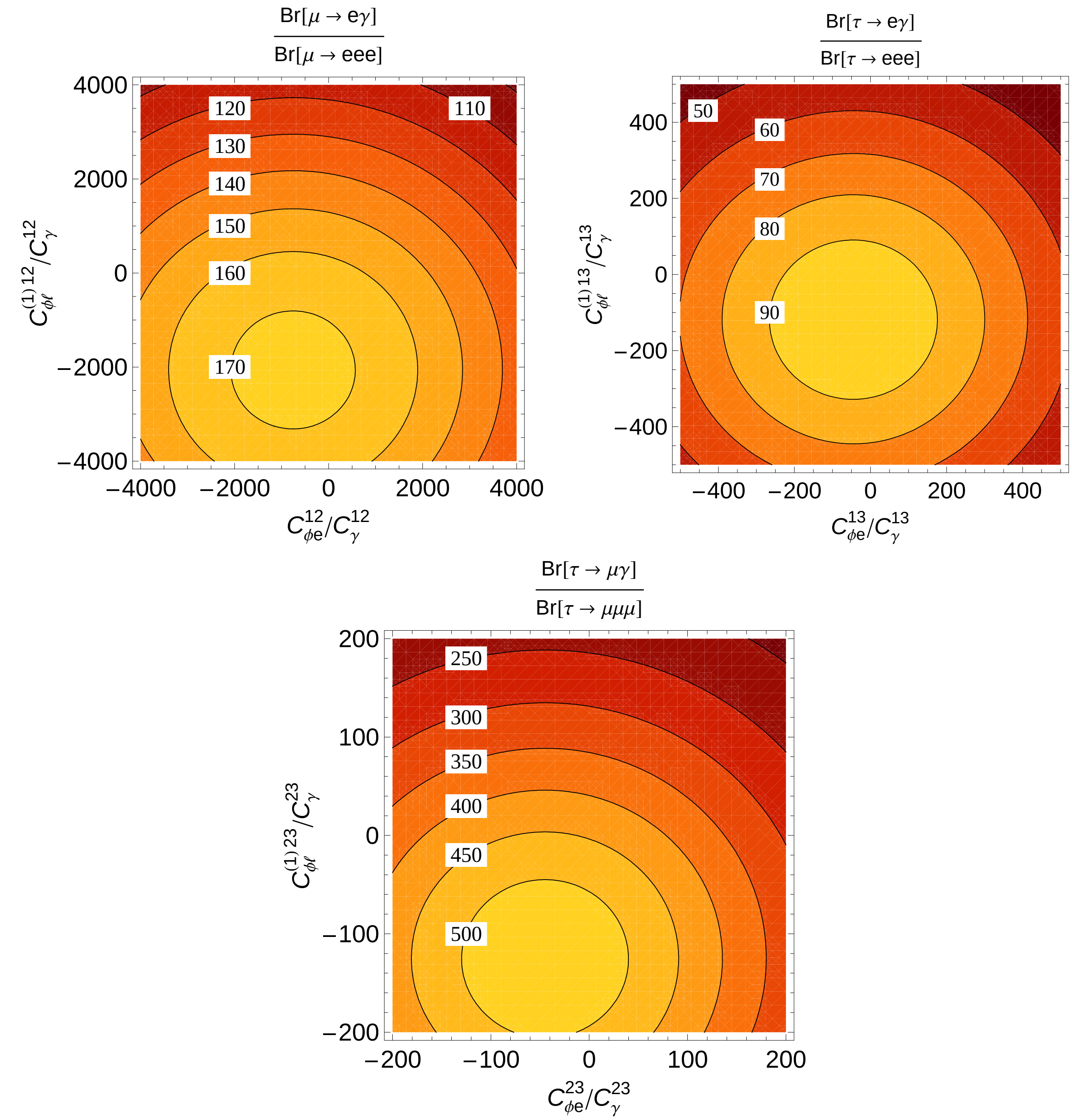


Figure 5.2: Ratios Br[$\ell_i \to \ell_f \gamma$]/Br[$\ell_i \to \ell_f \ell_f \bar{\ell}_f$] in the $\frac{C^{fi}_{\varphi e}}{C^{fi}_{\gamma}}$–$\frac{C^{(1)\,fi}_{\varphi \ell}}{C^{fi}_{\gamma}}$ plane (independent of the scale Λ of NP).

Chapter 6

$Z \to \ell_f^+ \ell_i^-$ decay

In this Chapter we calculate the branching ratios of $Z \to e\mu$, $Z \to e\tau$ and $Z \to \tau\mu$ decays in the extension of the SM with gauge invariant operators of dimension-6.

The decay of Z boson to lepton pair with different flavors have been studied in many papers, for example [89, 90, 91, 92]. In the extended SM with massive neutrinos mixing as observed in the neutrino oscillation search, the predictions for the Z decay to lepton pairs with different flavor are as usual very suppressed and non-observable. In the minimally extended SM with massive neutrinos, the rates can be estimated as[89]:

$$\begin{aligned} Br(Z \to \tau\mu) &\sim 10^{-54}, & (6.1)\\ Br(Z \to e\mu) \sim Br(Z \to e\tau) &\sim 4 \times 10^{-60} & (6.2)\end{aligned}$$

where the branching ratio for the Z decay is defined as a sum of charged lepton states [89]:

$$Br(Z \to \ell_f^{\pm} \ell_i^{\mp}) = \frac{\Gamma(Z \to \ell_f^{\pm}\ell_i^{\mp} + \ell_f^{\mp}\ell_i^{\pm})}{\Gamma_Z}, \tag{6.3}$$

with $\Gamma_Z \approx 2.495$ GeV being the total decay width of Z boson.

The current experimental bounds for the $Z \to ll'$ decays are collected in Table 6.1. These bounds are the best direct limits obtained by LEP 1 experiments. Planned GigaZ option of the TESLA Linear Collider project [93], if build, could increase the sensitivities to the LFV decays of the Z boson [94, 89] up to

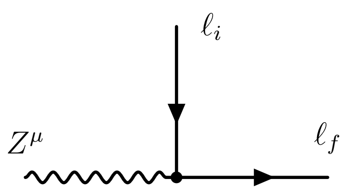
$$\begin{aligned} Br(Z \to e\mu) &\sim 2 \times 10^{-9},\\ Br(Z \to e\tau) &\sim k \times 6.5 \times 10^{-8},\\ BR(Z \to \tau\mu) &\sim k \times 2.2 \times 10^{-8}, \end{aligned} \tag{6.4}$$

Process	Experimental bound
$\mathrm{Br}\left[Z^0 \to \mu^\pm e^\mp\right]$	1.7×10^{-6} [92]
$\mathrm{Br}\left[Z^0 \to \tau^\pm e^\mp\right]$	9.8×10^{-6} [92]
$\mathrm{Br}\left[Z^0 \to \tau^\pm \mu^\mp\right]$	1.2×10^{-5} [92]

Table 6.1: Experimental upper limits (95 % CL) on the lepton flavor violating Z decay rates.

(with the factor k ranging from 0.2 to 1.0), giving the important new insight to the LFV violation in Z boson decays.

In the extended SM with gauge invariant operators of dimension-6, the LFV $Z \to \ell_f^+ \ell_i^-$ vertex exists already at the tree level:



$$i(\gamma^\mu\left[\Gamma_{fi}^{ZL} P_L + \Gamma_{fi}^{ZR} P_R\right] + i\sigma^{\mu\nu}[C_{fi}^{ZL} P_L + C_{fi}^{ZR} P_R] q_\nu)$$

$$\Gamma_{fi}^{ZL} = \tfrac{e}{2 s_W c_W}\big(\tfrac{v^2}{\Lambda^2}(C_{\phi l}^{(1)fi} + C_{\phi l}^{(3)fi}) + (1 - 2 s_W^2)\delta_{fi}$$

$$\Gamma_{fi}^{ZR} = \tfrac{e}{2 s_W c_W}\big(\tfrac{v^2}{\Lambda^2} C_{\phi e}^{fi} - 2 s_W^2 \delta_{fi}\big)$$

$$C_{fi}^{ZR} = C_{if}^{ZL*} = \tfrac{-v\sqrt{2}}{\Lambda^2}(s_W C_{eB}^{fi} + c_W C_{eW}^{fi}) \equiv \tfrac{-v\sqrt{2}}{\Lambda^2} C_Z^{fi}$$

The branching ratio for the lepton flavor violating decay $Z^0 \to \ell_f^- \ell_i^+$ is given by[1]:

$$\mathrm{Br}\left[Z^0 \to \ell_f^\pm \ell_i^\mp\right] = \frac{m_Z}{24\pi\Gamma_Z}\left[\frac{m_Z^2}{2}\left(\left|C_{fi}^{ZR}\right|^2 + \left|C_{fi}^{ZL}\right|^2\right) + \left|\Gamma_{fi}^{ZL}\right|^2 + \left|\Gamma_{fi}^{ZR}\right|^2\right], \qquad (6.5)$$

As in previous Chapters, normalizing the formulae for the branching ratios to the current experimental bound listed in Table 6.1 we derived the numerical equations constraining the

[1]As we calculate the branching ratios for the $Z \to \ell_f^+ \ell_i^-$ without summing over charges, our theoretical result must be multiplied by a factor 2 in order to compare with the experimental results of Table 6.1.

Wilson coefficients contributing to this decay:

$$\begin{aligned}
\sqrt{\left|C_{\varphi\ell}^{(1)12}+C_{\varphi\ell}^{(3)12}\right|^2+\left|C_{\varphi e}^{12}\right|^2+|C_Z^{12}|^2+|C_Z^{21}|^2} &\le 0.06\left(\frac{\Lambda}{1\text{ TeV}}\right)^2\sqrt{\frac{\text{Br}\left[Z^0\to\mu^\pm e^\mp\right]}{1.7\times10^{-6}}}, \\
\sqrt{\left|C_{\varphi\ell}^{(1)13}+C_{\varphi\ell}^{(3)13}\right|^2+\left|C_{\varphi e}^{13}\right|^2+|C_Z^{13}|^2+|C_Z^{31}|^2} &\le 0.14\left(\frac{\Lambda}{1\text{ TeV}}\right)^2\sqrt{\frac{\text{Br}\left[Z^0\to\tau^\pm e^\mp\right]}{9.8\times10^{-6}}}, \\
\sqrt{\left|C_{\varphi\ell}^{(1)23}+C_{\varphi\ell}^{(3)23}\right|^2+\left|C_{\varphi e}^{23}\right|^2+|C_Z^{23}|^2+|C_Z^{32}|^2} &\le 0.16\left(\frac{\Lambda}{1\text{ TeV}}\right)^2\sqrt{\frac{\text{Br}\left[Z^0\to\tau^\pm \mu^\mp\right]}{1.2\times10^{-5}}}.
\end{aligned} \tag{6.6}$$

It is interesting to note that although bounds on LFV Z decays are weaker then for LFV photon couplings, they put constraints on the linear combination $C_Z^{fi}=s_W C_{eB}^{fi}+c_W C_{eW}^{fi}$ which is "orthogonal" to $C_\gamma^{fi}=c_W C_{eB}^{IJ}-s_W C_{eW}^{IJ}$. Thus, using both constrains on C_γ^{fi} for the $\ell_i\to\ell_f\gamma$ decay and on C_Z^{fi} from Z decays one can independently constrain both C_{eW}^{fi} and C_{eB}^{fi}.

To illustrate the interplay between Wilson coefficients in the decays $\ell_i\to\ell_j\gamma$, $\ell_i\to\ell_j\ell_j\bar{\ell}_j$ and $Z\to\ell_f\ell_i$, let's consider their dependence on the Wilson coefficients $C_{\varphi e}^{ji}$ and C_{eW}^{ij}. The corresponding branching ratio for the $Z\to\ell_f\ell_i$ decay for this reduced case is given by:

$$Br(Z\to\ell_j\ell_i)=\frac{m_Z^3v^2}{24\pi\Gamma_Z\Lambda^4}[c_W^2|C_{eW}^{ji}|^2+|C_{\varphi e}^{ji}|^2]. \tag{6.7}$$

For the $\ell_i\to\ell_j\ell_j\bar{\ell}_j$ decays relevant expression reads as:

$$\begin{aligned}
Br(\ell_i\to\ell_j\ell_j\bar{\ell}_j) = \frac{m_i^5}{12288\pi^3\Lambda^4\Gamma_i}&[(16s_W^2+5(2s_W^2-1)^2)|C_{\varphi e}|^2 \\
-\frac{32\sqrt{2}ev(6s_W^2-1)c_W}{m_i}Re[C_{\varphi e}^{ij}C_{eW}^{ij}]+\frac{256e^2v^2c_W^2}{m_i^2}&(\log\frac{m_i^2}{m_j^2}-\frac{11}{4})|C_{eW}^{ij}|^2],
\end{aligned} \tag{6.8}$$

and finally for the $\ell_i\to\ell_j\gamma$ decay it is:

$$\begin{aligned}
Br(\ell_i\to\ell_j\gamma) &= \frac{m_i^3}{16\pi\Lambda^4\Gamma_i}\left[\left|\frac{4em_i}{3(4\pi)^2}(\frac{3}{2}-s_W^2)C_{\varphi e}^{ji}+v\sqrt{2}s_W C_{eW}^{ij}\right|^2\right. \\
&+ \left.\left|\frac{4em_j}{3(4\pi)^2}(\frac{3}{2}-s_W^2)C_{\varphi e}^{ji}+v\sqrt{2}s_W C_{eW}^{ij}\right|^2\right].
\end{aligned} \tag{6.9}$$

The allowed regions in $C_{eW}^{ij}-C_{\varphi e}^{ji}$ planes for the scale of new physics $\Lambda=10$ TeV are shown in Fig. 6.1. As we see, the plots confirm that tree level photon couplings are much more strongly constrained by the radiative lepton decays then by 3-body decays, and in

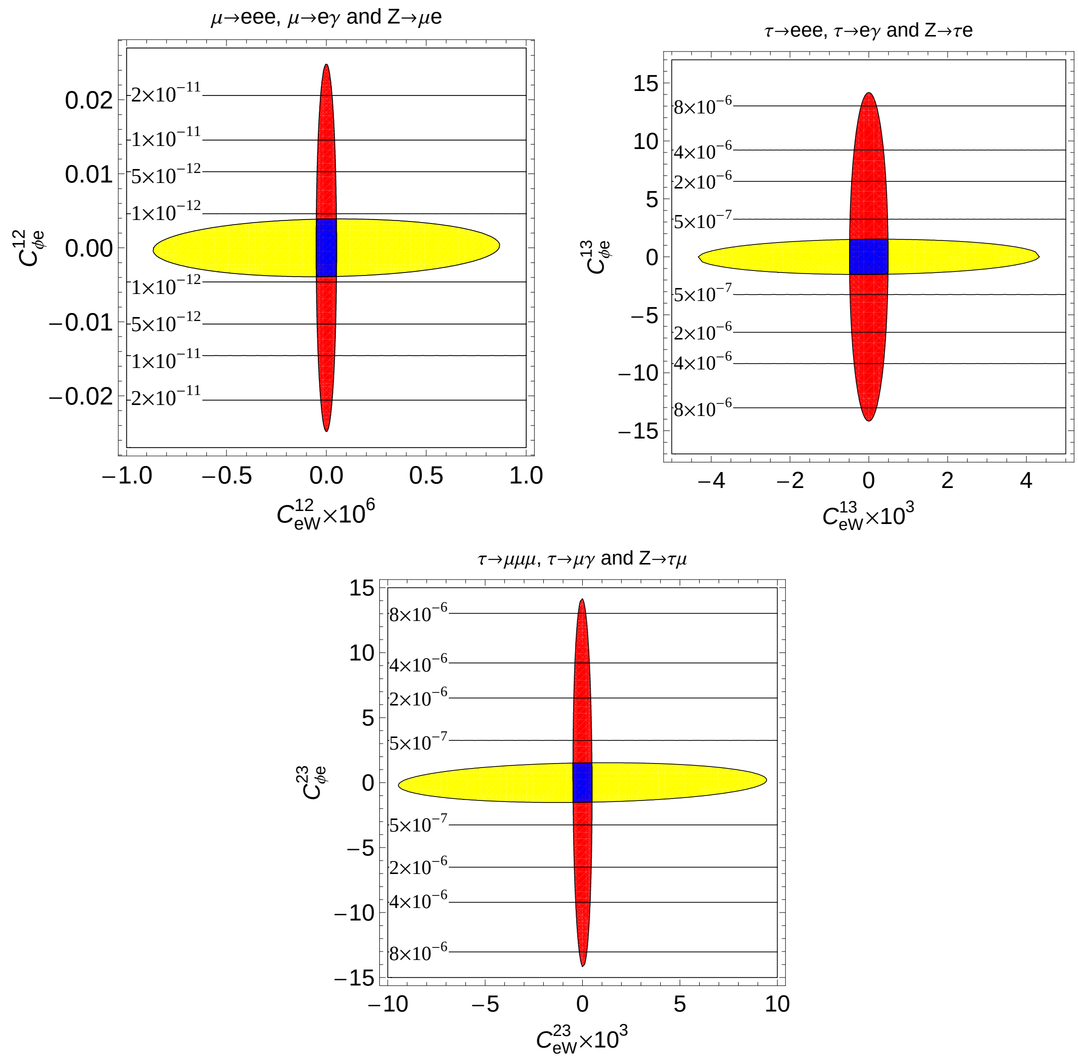


Figure 6.1: Allowed regions in the C_{eW}^{fi}–$C_{\varphi e}^{fi}$ plane for $\Lambda = 10$ TeV. Yellow (lightest): $\ell_i \to \ell_f \ell_f \bar{\ell}_f$, red (gray): $\ell_i \to \ell_f \gamma$. The blue region is allowed by both decay modes simultaneously. The contour lines show the predicted branching ratio for $Z \to \ell_f \ell_i$.

general are quite tight, in particular for $\mu \to e\gamma$ decay. The dependence of LFV Z decays on C_{eW} is weak in the range allowed by other decays.

We can also obtain absolute upper bounds on the coefficients C_{eB} and C_{eW} by using the decay of photon and Z boson to lepton pairs. Let's consider the decay of $\tau \to \mu\gamma$ and $Z \to \tau\mu$ as an example. The expression for the decay of $\tau \to \mu\gamma$ reads as:

$$Br(\tau \to \mu\gamma) = \frac{m_\tau^3}{4\pi\Lambda^4\Gamma_\tau} |c_W C_{eB}^{23} - s_W C_{eW}^{23}|^2. \tag{6.10}$$

The branching ratio for the decay of $Z \to \tau\mu$ is given by:

$$Br(Z \to \tau\mu) = \frac{m_Z^3 v^2}{48\pi\Lambda^2\Gamma_Z}|s_W C_{eB}^{23} + c_W C_{eW}^{23}|^2. \tag{6.11}$$

Excluded area for the scale of New Physics $\Lambda = 1$ TeV is shown in Fig. 6.2. The bound on radiative decay $\tau \to \mu\gamma$ strongly correlates the allowed values for C_{eB} and C_{eW} to a thin straight belt, while $Z \to \tau\mu$ bound cuts the length of this belt to a wide but finite compartment.

Concerning flavor diagonal transitions we can correlate the anomalous magnetic moments or electric dipole moments to Z decay to lepton pairs and constrain the flavor-diagonal coefficients C_{eW}^{ii} and C_{eB}^{ii}. Hear as an example we show the bounds given by anomalous magnetic moments. For the electron and muon constraints from the anomalous magnetic moments are so strong that no sizable effects of new physics in $Z \to ee$ and $Z \to \mu\mu$ are possible. However, for the tau lepton the constraints on NP generated for anomalous magnetic moment of taon and decay of $Z \to \tau\tau$ are not much different. The anomalous magnetic moment of taon reads as:

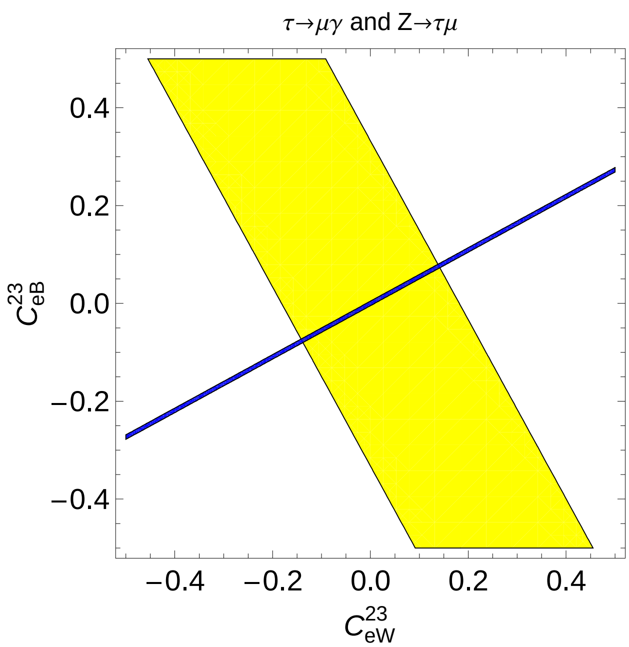


Figure 6.2: Allowed regions from Br[$Z^0 \to \tau\mu$] (yellow) and Br[$\tau \to \mu\gamma$] (blue) in the C_{eW}^{23}–C_{eB}^{23} plane for $\Lambda = 1$ TeV.

$$a_\tau = \frac{2m_\tau v\sqrt{2}}{e\Lambda^2} Re[c_W C_{eB}^{33} - s_W C_{eW}^{33}]. \tag{6.12}$$

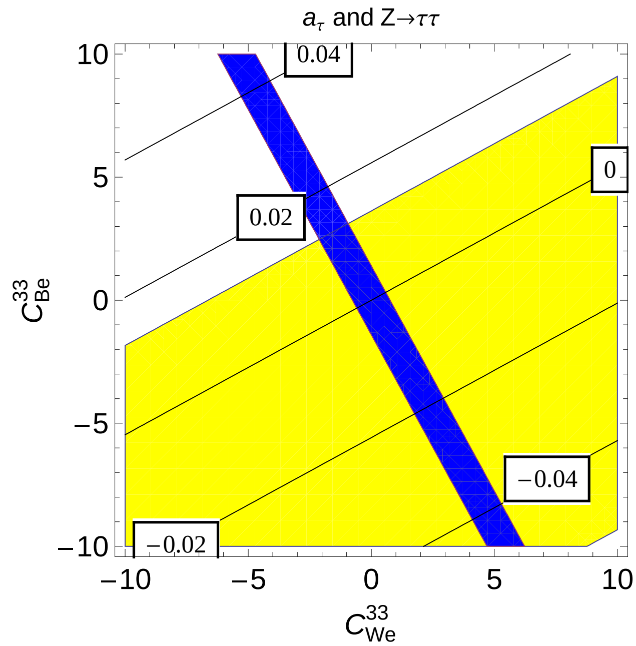


Figure 6.3: Correlations between the anomalous magnetic moment of the τ lepton and $Z^0 \to \tau\tau$. Yellow (light grey): region allowed by the a_τ, blue (dark grey): region allowed by $Z^0 \to \tau\tau$. The contour lines indicate the value of a_τ for $\Lambda = 1$ TeV.

The expression for branching ratio of $Z \to \tau\tau$ is:

$$Br(Z \to \tau\tau) = \frac{v^2 m_Z}{24\Lambda^2\pi\Gamma_Z}[\frac{m_Z^2}{2}|s_W C_{eB}^{33} + c_W C_{eW}^{33}|^2 + \frac{e^2(1-2s_W^2)^2}{4s_W^2 c_W^2} + \frac{e^2 s_W^2}{c_W^2}]. \tag{6.13}$$

To obtain the bounds we use $Br(Z \to \tau\tau) = 3.370 \pm 0.008\%$ [18] (multiplying it by a correction factor $Br(Z \to \tau\tau)_{SM}/Br(Z \to \tau\tau)_{tree-level}$) and assumed that the contributions from C_{eB}^{33}, C_{eW}^{33} in eq. (6.13) are constrained by the error of $Z \to \tau\tau$ width measurement and by the taon AMM. Then the allowed region in the $C_{eW}^{33} - C_{eW}^{33}$ is shown in Fig. 6.3 (assuming the scale of NP $\Lambda = 1$ TeV).

Chapter 7

Summary and conclusions

The Standard Model of elementary particle interactions proved to be an overwhelming success - more than 40 years since its creation, in principle all measurements done in laboratories on Earth confirm its predictions (assuming we that we consider SM version extended with massive neutrinos). However, various theoretical problems - scalar mass hierarchy problem, imperfect gauge coupling unification and no good explanation of observed pattern of SM flavor structure and fermion masses, to name just the most important ones - lead to common believe of most of the high energy physicists that in spite of its success the Standard Model is only an effective approximation of some more fundamental theory. This believe is confirmed also by cosmological observations - matter-antimatter asymmetry in the Universe (connected to CP violation), presence of large amounts of dark matter or dark energy cannot be explained in the frame of SM.

Flavor physics may give us a very important insight how to construct a more fundamental theory of elementary particle interactions. In particular observation of processes of lepton number and/or CP violation could be very enlightening. In the minimal SM lepton flavor numbers are strictly conserved. Also in the extension of the SM with massive neutrinos, the branching ratios of charged lepton flavor violating decays are too small to be observed observable in the experiments, currents or future. However, many SM extensions does predict the flavor violation in the charged lepton sector. An observation of the charged LFV processes at experiment would be a clear hint for the physics beyond the SM. Thus, currently serious efforts are being made to design new experiments or upgrade existing ones to reach higher sensitivities to such processes.

In this thesis, we investigate the most important lepton flavor observables in the SM extended with the most general form of gauge invariant dimension-6 operators. In such approach possible New Physics effects are parametrized by the full basis of the gauge invariant operators of mass dimension-5 and -6, constructed from the fields of the Standard Model. We calculated the expression for several theoretically important and experimentally well constrained lepton flavor observables, giving for them the compact results in terms of Wilson coefficients of all effective operators which could give contributions to such processes. In particular, we computed the complete set of the tree-level and one loop contributions to the radiative lepton decays $l \to l'\gamma$, as well as to lepton Electric Dipole Moments and Anomalous Magnetic Moments. We also obtained the formulae for all variants of the three body charged lepton decay rates and for the flavor violating Z boson decays to lepton pairs - in both those cases taking into account all possible tree level contributions.

The derived expressions allowed us to obtain model independent bounds on the Wilson coefficients of LFV operators (or rather on their combinations). Having such bounds can significantly simplify and speed up the comparison with the experiment the specific models of New Physics, as now only the predicted by them values of Wilson coefficients need to be calculated in terms of parameters of such models.

We used our results to illustrate the typical size of constraints on Wilson coefficients related to LFV in the charged lepton sector. Such bounds scale with the square of the New Physics scale, so decrease like $1/\Lambda^2$. If Λ is low (in $\mathcal{O}(1)$ TeV range accessible at the LHC), bounds are very strong, especially for the transitions between 2nd and 1st generation. Thus, if there is any New Physics in the TeV range, already existing experiments show us that it has to be extremely weakly coupled to the SM charged lepton sector.

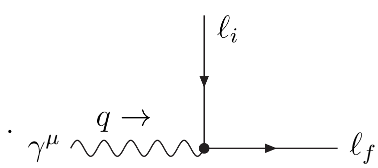
Apart from obtaining the general magnitude of bounds on Wilson coefficients, we discussed also several examples how combining various processes could help us finding correlations (or cancellations) between different LFV couplings. Such correlations can be potentially very important for designing new experiments. For example we have shown that additional contributions could change even by factor of 2 the ratio of $Br(l \to l'\gamma)/Br(l \to 3l')$, fixed in the scenario of photon-penguin domination. Such information could help experimentalists at the PSI institute to decide it is worth to search independently for both $\mu \to e\gamma$ and $\mu \to eee$ decays or concentrate efforts on upgrading sensitivity for the more promising of them.

Appendix A

Feynman rules for SM with operators of dimension-6

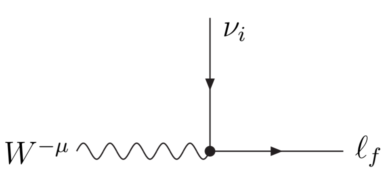
In this Appendix we collect the Feynman rules for the lepton couplings in the SM extended with operators of dimension-6. We list only the couplings which we used in our analysis (so for example we do not include here couplings of the physical Higgs boson to leptons). For completeness, we give below also few useful purely SM couplings, like $W^+W^-\gamma$ or Goldstone-γ vertices.

A.0.1 Feynman rules involving gauge and Goldstone bosons



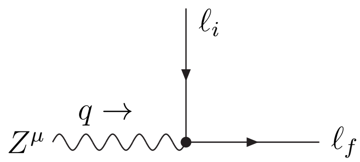
$$i\left(e\gamma^\mu\delta^{fi} + i\sigma^{\mu\nu}\left[C^{fi}_{\gamma L}P_L + C^{fi}_{\gamma R}P_R\right]q_\nu\right)$$

$$C^{\gamma R}_{fi} = C^{\gamma L\star}_{fi} = \frac{\upsilon\sqrt{2}}{\Lambda^2}\left(c_W C^{fi}_{eB} - s_W C^{fi}_{eW}\right)$$



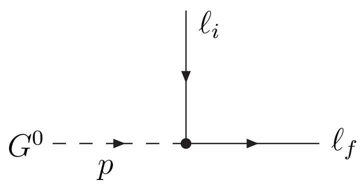
$$i\Gamma^{WL}_{fj}V^{PMNS}_{ji}\gamma^\mu P_L$$

$$\Gamma^{WL}_{fj} = -\frac{e}{\sqrt{2}s_W}\left(\frac{v^2}{\Lambda^2}C^{(3)fj}_{\phi\ell} + \delta_{fj}\right)$$



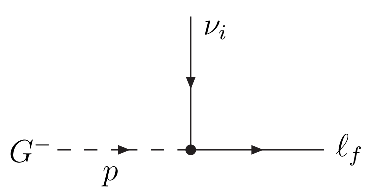
$$i\left(\gamma^\mu\left[\Gamma^{ZL}_{fi}P_L+\Gamma^{ZR}_{fi}P_R\right]+i\sigma^{\mu\nu}\left[C^{ZL}_{fi}P_L+C^{ZR}_{fi}P_R\right]q_\nu\right)$$

$$\begin{aligned}
\Gamma^{ZL}_{fi} &= \frac{e}{2s_Wc_W}\left(\frac{v^2}{\Lambda^2}\left(C^{(1)fi}_{\phi\ell}+C^{(3)fi}_{\phi\ell}\right)+\left(1-2s_W^2\right)\delta_{fi}\right)\\
\Gamma^{ZR}_{fi} &= \frac{e}{2s_Wc_W}\left(\frac{v^2}{\Lambda^2}C^{fi}_{\phi e}-2s_W^2\delta_{fi}\right)\\
C^{ZR}_{fi} &= C^{ZL\star}_{if} = -\frac{v\sqrt{2}}{\Lambda^2}\left(s_WC^{fi}_{eB}+c_WC^{fi}_{eW}\right)
\end{aligned}$$



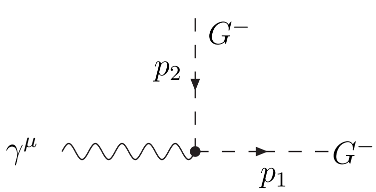
$$\begin{aligned}
&-\left(\left(\not{p}\Gamma^{G^0L}_{fi}+\frac{1}{v}\delta_{fi}m_{\ell_i}\right)P_L\right.\\
&\left.+\left(\not{p}\Gamma^{G^0R}_{fi}-\frac{1}{v}\delta_{fi}m_{\ell_i}\right)P_R\right)
\end{aligned}$$

$$\begin{aligned}
\Gamma^{G^0L}_{fi} &= \frac{v}{\Lambda^2}\left(C^{(1)fi}_{\phi\ell}+C^{(3)fi}_{\phi\ell}\right)\\
\Gamma^{G^0R}_{fi} &= \frac{v}{\Lambda^2}C^{fi}_{\phi e}
\end{aligned}$$

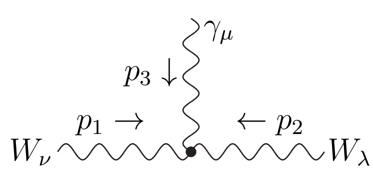


$$i\left(\Gamma^{G^-L}_{fj}\not{p}-\frac{\sqrt{2}}{v}\delta_{fj}m_{\ell_f}\right)V^{PMNS}_{ji}P_L$$

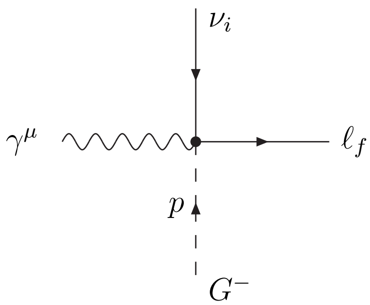
$$\Gamma^{G^-L}_{fj} = -\frac{v\sqrt{2}}{\Lambda^2}C^{(3)fj}_{\phi l}$$



$$ie\left(p_1^\mu+p_2^\mu\right)$$

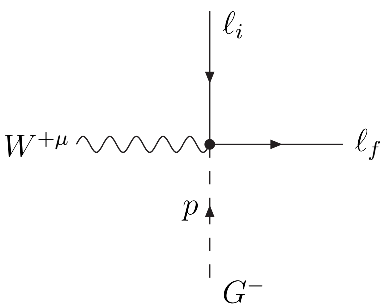


$$ie[g^{\nu\lambda}(p_1-p_2)^\mu+g^{\lambda\mu}(p_2-p_3)^\nu+g^{\mu\nu}(p_3-p_1)^\lambda]$$



$$i\Gamma^{G\gamma L}_{fj} V^{PMNS}_{ji} \gamma^\mu P_L$$

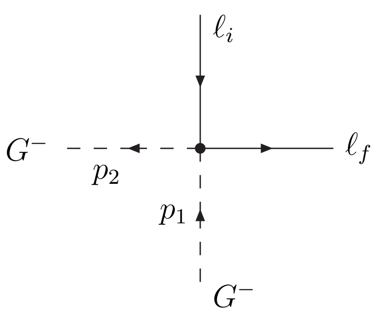
$$\Gamma^{G\gamma L}_{fj} = -\frac{ev\sqrt{2}}{\Lambda^2} C^{(3)fj}_{\phi l}$$



$$i\gamma^\mu \left[\Gamma^{GWL}_{fi} P_L + \Gamma^{GWR}_{fi} P_R\right]$$

$$\Gamma^{GWL}_{fi} = -\frac{ev}{\Lambda^2 s_W} C^{(1)fi}_{\phi\ell}$$

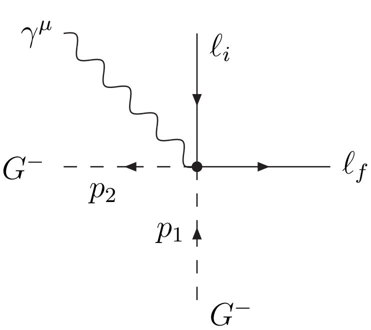
$$\Gamma^{GWR}_{fi} = -\frac{ev}{\Lambda^2 s_W} C^{fi}_{\phi e}$$



$$i\left(\not{p}_1 + \not{p}_2\right) \left[\Gamma^{GGL}_{fi} P_L + \Gamma^{GGR}_{fi} P_R\right]$$

$$\Gamma^{GGL}_{fi} = -\frac{1}{\Lambda^2} \left(C^{(1)fi}_{\phi l} - C^{(3)fi}_{\phi\ell}\right)$$

$$\Gamma^{GGR}_{fi} = -\frac{1}{\Lambda^2} C^{fi}_{\phi e}$$

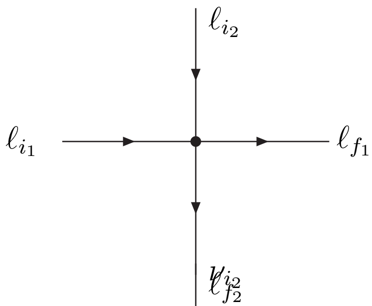


$$i\gamma^\mu \left(\Gamma^{GG\gamma L}_{fi} P_L + \Gamma^{GG\gamma R}_{fi} P_R\right)$$

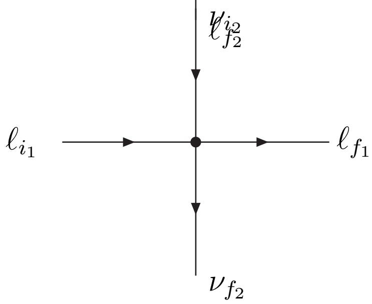
$$\Gamma^{GG\gamma L}_{fi} = -\frac{2e}{\Lambda^2} \left(C^{(1)fi}_{\phi\ell} - C^{(3)fi}_{\phi\ell}\right)$$

$$\Gamma^{GG\gamma R}_{fi} = -\frac{2e}{\Lambda^2} C^{fi}_{\phi e}$$

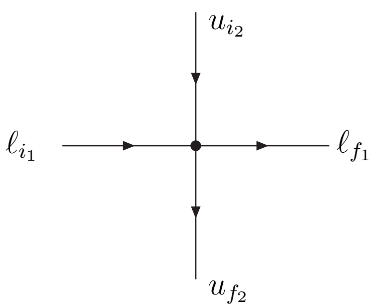
A.0.2 Feynman rules for 4-fermion operators



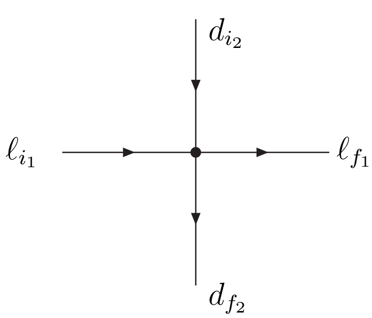
$$\frac{i}{\Lambda^2}\Big[C_{\ell\ell}^{f_1i_1f_2i_2}(\gamma^\mu P_L)_{f_1i_1}(\gamma_\mu P_L)_{f_2i_2}$$
$$+C_{ee}^{f_1i_1f_2i_2}(\gamma^\mu P_R)_{f_1i_1}(\gamma_\mu P_R)_{f_2i_2}$$
$$+C_{\ell e}^{f_1i_1f_2i_2}(\gamma^\mu P_L)_{f_1i_1}(\gamma_\mu P_R)_{f_2i_2}\Big]$$



$$\frac{i}{\Lambda^2}\Big[C_{\ell\ell}^{f_1i_1f_2i_2}(\gamma^\mu P_L)_{i_1f_1}(\gamma_\mu P_L)_{f_2i_2}$$
$$+2\mathrm{Re}(C_{\ell e}^{f_1i_1f_2i_2})(\gamma^\mu P_L)_{f_1i_1}(\gamma_\mu P_R)_{f_2i_2}\Big]$$



$$\frac{i}{\Lambda^2}\Big[(C_{\ell q}^{(1)f_1i_1f_2i_2}-C_{\ell q}^{(3)f_1i_1f_2i_2})(\gamma^\mu P_L)_{i_1f_1}(\gamma_\mu P_L)_{f_2i_2}$$
$$+C_{\ell u}^{f_1i_1f_2i_2}(\gamma^\mu P_L)_{f_1i_1}(\gamma_\mu P_R)_{f_2i_2}+C_{eq}^{f_1i_1f_2i_2}(\gamma^\mu P_R)_{f_1i_1}(\gamma_\mu P_L)_{f_2i_2}$$
$$+C_{eu}^{f_1i_1f_2i_2}(\gamma^\mu P_R)_{f_1i_1}(\gamma_\mu P_R)_{f_2i_2}$$
$$-C_{\ell equ}^{(1)f_1i_1f_2i_2}(P_R)_{f_1i_1}(P_R)_{f_2i_2}-C_{\ell equ}^{(1)i_1f_1i_2f_2\star}(P_L)_{f_1i_1}(P_L)_{f_2i_2}$$
$$-C_{\ell equ}^{(3)f_1i_1f_2i_2}(\sigma^{\mu\nu}P_R)_{f_1i_1}(\sigma_{\mu\nu}P_R)_{f_2i_2}$$
$$-\,C_{\ell equ}^{(3)i_1f_1i_2f_2\star}(\sigma^{\mu\nu}P_L)_{f_1i_1}(\sigma_{\mu\nu}P_L)_{f_2i_2}\Big]$$



$$\frac{i}{\Lambda^2}\Big[(C_{\ell q}^{(1)f_1i_1f_2i_2}+C_{\ell q}^{(3)f_1i_1f_2i_2})(\gamma^\mu P_L)_{i_1f_1}(\gamma_\mu P_L)_{f_2i_2}$$
$$+C_{\ell d}^{f_1i_1f_2i_2}(\gamma^\mu P_L)_{f_1i_1}(\gamma_\mu P_R)_{f_2i_2}+C_{eq}^{f_1i_1f_2i_2}(\gamma^\mu P_R)_{f_1i_1}(\gamma_\mu P_L)_{f_2i_2}$$
$$+C_{ed}^{f_1i_1f_2i_2}(\gamma^\mu P_R)_{f_1i_1}(\gamma_\mu P_R)_{f_2i_2}$$
$$+\,C_{\ell edq}^{f_1i_1f_2i_2}(P_R)_{f_1i_1}(P_L)_{f_2i_2}+C_{\ell edq}^{i_1f_1i_2f_2\star}(P_L)_{f_1i_1}(P_R)_{f_2i_2}\Big]$$

Appendix B

The decomposition and expansion of the loop functions

In our calculation we have used the loop functions in the standard Passarino-Veltman decomposition [95]. The basic ("master") 1-, 2- and 3- loop functions are defined (in the momenta conventions which we used for diagram calculations) as follows:

$$\mu^{4-d}\int\frac{d^dk}{(2\pi)^d}\frac{1}{k^2-m^2} = \frac{i}{(4\pi)^2}a_0(m^2), \tag{B.1}$$

$$\mu^{4-d}\int\frac{d^dk}{(2\pi)^d}\frac{1}{[k^2-m_1^2][(k-q)^2-m_2^2]} = \frac{i}{(4\pi)^2}b_0(q,m_1^2,m_2^2), \tag{B.2}$$

$$\int\frac{d^dk}{(2\pi)^d}\frac{1}{[k^2-m_1^2][(k-q)^2-m_2^2][(k-p)^2-m_3^2]} = \frac{i}{(4\pi)^2}c_0(q,p,m_1^2,m_2^2,m_3^2). \tag{B.3}$$

Due to complicated structure of dimension-6 vertices, in our calculations we had to include tensor 1-loop integrals with maximum 4 powers of loop momentum in the numerator. However, 4th order integrals always appeared as $k^\alpha k^\beta k^2$ and could have been easily reduced to lower integrals using identity $k^2/(k^2-m^2)\to 1+m^2/(k^2-m^2)$. Remaining integrals could be expressed as (we do not write explicitly the arguments of b_{ij}, c_{ij} functions, they are the same as in the equations above):

$$\mu^{4-d}\int\frac{d^dk}{(2\pi)^d}\frac{k^\mu}{[k^2-m_1^2][(k-q)^2-m_2^2]}=\frac{i}{(4\pi)^2}q^\mu b_1, \tag{B.4}$$

$$\mu^{4-d}\int\frac{d^dk}{(2\pi)^d}\frac{k^\mu k^\nu}{[k^2-m_1^2][(k-q)^2-m_2^2]}=\frac{i}{(4\pi)^2}\left(q^\mu q^\nu b_{21}+g^{\mu\nu}b_{22}\right), \tag{B.5}$$

$$\mu^{4-d}\int\frac{d^dk}{(2\pi)^d}\frac{k^\mu k^\nu k^\alpha}{[k^2-m_1^2][(k-q)^2-m_2^2]}=\frac{i}{(4\pi)^2}\left(q^\mu q^\nu q^\alpha b_{31}+(g^{\mu\nu}q^\alpha+g^{\mu\alpha}q^\mu+g^{\alpha\nu}q^\mu)b_{32}\right). \tag{B.6}$$

$$\mu^{4-d}\int\frac{d^dk}{(2\pi)^d}\frac{k^\mu}{[k^2-m_1^2][(k-q)^2-m_2^2][(k-p)^2-m_3^2]}=\frac{i}{(4\pi)^2}\left(q^\mu c_{11}+p^\mu c_{12}\right), \tag{B.7}$$

$$\mu^{4-d}\int\frac{d^dk}{(2\pi)^d}\frac{k^\mu k^\nu}{[k^2-m_1^2][(k-q)^2-m_2^2][(k-p)^2-m_3^2]}$$
$$=\frac{i}{(4\pi)^2}\left(q^\mu q^\nu c_{21}+p^\mu p^\nu c_{22}+(p^\mu q^\nu+p^\nu q^\mu)c_{23}+g^{\mu\nu}\right), \tag{B.8}$$

$$\mu^{4-d}\int\frac{d^dk}{(2\pi)^d}\frac{k^\mu k^\nu}{[k^2-m_1^2][(k-q)^2-m_2^2][(k-p)^2-m_3^2]}$$
$$=\frac{i}{(4\pi)^2}\left(q^\mu q^\nu c_{21}+p^\mu p^\nu c_{22}+(p^\mu q^\nu+p^\nu q^\mu)c_{23}+g^{\mu\nu}c_{24}\right), \tag{B.9}$$

$$\mu^{4-d}\int\frac{d^dk}{(2\pi)^d}\frac{k^\mu k^\nu k^\alpha}{[k^2-m_1^2][(k-q)^2-m_2^2][(k-p)^2-m_3^2]}$$
$$\begin{aligned}&=\frac{i}{(4\pi)^2}\big(p^\mu p^\nu p^\alpha c_{31}+q^\mu q^\nu q^\alpha c_{32}+c_{33}(p^\mu g^{\alpha\beta}+p^\alpha g^{\mu\beta}+p^\beta g^{\mu\alpha})\\&+c_{34}(q^\mu g^{\alpha\beta}+q^\alpha g^{\mu\beta}+q^\beta g^{\mu\alpha})+c_{35}(p^\mu q^\alpha q^\beta+p^\alpha q^\mu q^\beta+p^\beta q^\mu q^\alpha)\\&+c_{36}(q^\mu p^\alpha p^\beta+q^\alpha p^\beta p^\mu+q^\beta p^\mu p^\alpha)\big).\end{aligned} \tag{B.10}$$

By algebraic manipulations tensor integrals can be reduced to combinations of "master integrals" a_0, b_0, c_0. The general form of resulting expressions is rather complicated, particularly for higher powers of loop momentum in the numerator. We performed such calculations with the use of Mathematica program. Here we list explicitly only the lower C_{1x} an C_{2x} tensor 3-point functions for the special case $m_1 = m_2$, applicable in the interesting us flavor-diagonal photon couplings.

In the equations below we suppress arguments of c-functions, they all should be understood as $c_{ij} = c_{ij}(q, p, , m_1, m_1, m_2)$. We also define function f as the finite part of b_0 function:

$$b_0(p, m_1, m_2) = \frac{2}{4-d}+1-\log\frac{m_1m_2}{\mu^2}+\frac{(m_1^2+m_2^2)\log\frac{m_2}{m_1}}{m_1^2-m_2^2}+f(p^2, m_1, m_2). \tag{B.11}$$

Then:

$$
\begin{aligned}
c_{11} &= \frac{1}{2(pq^2 - p^2q^2)} \left(pq \left(1 + \frac{2m_2^2 \log \frac{m_2}{m_1}}{m_1^2 - m_2^2} \right) \right. \\
&+ p^2 f(p^2, m_1, m_2) - pqf(q^2, m_1, m_1) - (p^2 - pq) f((p-q)^2, m_1, m_2) \\
&+ \left. ((m_1^2 - m_2^2 + p^2)pq - p^2q^2)c_0 \right), \qquad \text{(B.12)} \\
c_{12} &= \frac{1}{2(pq^2 - p^2q^2)} \left(-q^2 \left(1 + \frac{2m_2^2 \log \frac{m_2}{m_1}}{m_1^2 - m_2^2} \right) \right. \\
&- pqf(p^2, m_1, m_2) + q^2 f(q^2, m_1, m_1) + (pq - q^2) f((p-q)^2, m_1, m_2) \\
&+ \left. (-m_1^2 + m_2^2 - p^2 + pq)q^2 c_0 \right), \qquad \text{(B.13)} \\
c_{21} &= \frac{1}{8(pq^2 - p^2q^2)^2} \big(\big(2(m_1^2 - m_2^2 + 2p^2)(pq)^2 + 2(pq)^3 \\
&+ (m_1^2 - m_2^2 - p^2 - 5pq)p^2q^2 \big) \left(1 + \frac{2m_2^2 \log \frac{m_2}{m_1}}{m_1^2 - m_2^2} \right) \\
&+ 3p^2((m_1^2 - m_2^2 + p^2)pq - p^2q^2) f(p^2, m_1, m_2) \\
&- (2(m_1^2 - m_2^2 + p^2)(pq)^2 + 2(pq)^3 + p^2q^2(m_1^2 - m_2^2 + p^2 - 5pq)) f(q^2, m_1, m_1) \\
&+ ((p^2)^3(-3pq + 4q^2) + 2(pq)^2(-2(pq)^2 + (m_1^2 - m_2^2)q^2 \\
&+ pq(-3m_1^2 + 3m_2^2 + q^2)) + (p^2)^2(8(pq)^2 + pq(-3m_1^2 + 3m_2^2 - 16q^2) \\
&+ q^2(-m_1^2 + m_2^2 + 4q^2)) + p^2(-2(pq)^3 + pq(-3m_1^2 + 3m_2^2 - 5q^2)q^2 \\
&+ (m_1^2 - m_2^2)(q^2)^2 + 2(pq)^2(5m_1^2 - 5m_2^2 + 6q^2))) \frac{f((p-q)^2, m_1, m_2)}{(p-q)^2} \\
&+ (2((m_1^2 - m_2^2)^2 + (4m_1^2 - 2m_2^2)p^2 + (p^2)^2)(pq)^2 - 6pqp^2q^2(m_1^2 - m_2^2 + p^2) \\
&+ p^2q^2((m_1^2 - m_2^2)^2 + (p^2)^2 + p^2(-2(m_1^2 + m_2^2) + 3q^2)))c_0), \qquad \text{(B.14)} \\
c_{22} &= \frac{1}{8(pq^2 - p^2q^2)^2} (q^2(2(pq)^2 + (3m_1^2 - 3m_2^2 + p^2)q^2 - 3pqq^2) \\
&- \frac{6m_2^2 \log \frac{m_2}{m_1} (-m_1^2 + m_2^2 - p^2 + pq)(q^2)^2}{m_1^2 - m_2^2} \\
&- (2(m_1^2 - m_2^2 + p^2)(pq)^3 - 5p^2q^2pq(m_1^2 - m_2^2 + p^2) + 2p^2(pq)^2q^2 \\
&+ (p^2)^2(q^2)^2) \frac{f(p^2, m_1, m_2)}{p^2} + 3(-m_1^2 + m_2^2 - p^2 + pq)(q^2)^2 f(q^2, m_1, m_1) \\
&+ (-4(pq)^4 + 2(pq)^3(m_1^2 - m_2^2 + p^2 - q^2) + 4(pq)^2q^2(m_1^2 - m_2^2 + 3p^2 + 2q^2) \\
&+ (q^2)^2(4(p^2)^2 + 3(m_1^2 - m_2^2)q^2 + p^2(5m_1^2 - 5m_2^2 + 4q^2)) \\
&- q^2pq(5(p^2)^2 + 3q^2(3m_1^2 - 3m_2^2 + q^2) + p^2(5m_1^2 - 5m_2^2 + 16q^2))) \frac{f((p-q)^2, m_1, m_2)}{(p-q)^2} \\
&+ q^2(-6(m_1^2 - m_2^2 + p^2)pqq^2 + 2(pq)^2(2m_1^2 + q^2) + q^2(3(m_1^2 - m_2^2)^2 \\
&+ 3(p^2)^2 + p^2(2m_1^2 - 6m_2^2 + q^2)))c_0), \qquad \text{(B.15)}
\end{aligned}
$$

$$
\begin{aligned}
c_{23} &= \frac{1}{8(pq^2-p^2q^2)^2}(-2(pq)^3-(3m_1^2-3m_2^2+p^2)pqq^2+(pq)^2q^2+2p^2(q^2)^2 \\
&+ \frac{2m_2^2\log\frac{m_2}{m_1}q^2(-3(m_1^2-m_2^2+p^2)pq+(pq)^2+2p^2q^2)}{m_1^2-m_2^2} \\
&- ((m_1^2-m_2^2+p^2)(pq)^2+2p^2(m_1^2-m_2^2+p^2)q^2-3p^2pqq^2)f(p^2,m_1,m_2) \\
&- q^2(-3(m_1^2-m_2^2+p^2)pq+(pq)^2+2p^2q^2)f(q^2,m_1,m_1) \\
&+ (2(p^2)^3q^2+pq(3(-m_1^2+m_2^2)(q^2)^2+pqq^2(9m_1^2-9m_2^2+q^2) \\
&- 2(pq)^2(2m_1^2-2m_2^2+q^2))+(p^2)^2((pq)^2-10q^2pq \\
&+ 2q^2(m_1^2-m_2^2+2q^2))+p^2(-2(pq)^3+2(q^2)^3-5q^2pq(m_1^2-m_2^2+2q^2) \\
&+ (pq)^2(m_1^2-m_2^2+14q^2)))\frac{f((p-q)^2,m_1,m_2)}{(p-q)} \\
&+ (-4m_1^2(pq)^3+4(m_1^2-m_2^2+p^2)(pq)^2q^2+2p^2(m_1^2-m_2^2+p^2)(q^2)^2 \\
&- q^2pq(3(m_1^2-m_2^2)^2+3(p^2)^2+p^2(2m_1^2-6m_2^2+3q^2)))c_0) \qquad \text{(B.16)} \\
c_{24} &= \frac{1}{4}\left(\frac{2}{4-d}-\log\frac{m_1m_2}{\mu^2}\right)+\frac{1}{8(pq^2-p^2q^2)}(4(pq)^2+(m_1^2-m_2^2-3p^2-pq)q^2 \\
&+ \frac{2\log\frac{m_2}{m_1}((m_1^2+m_2^2)(pq)^2+(m_1^2m_2^2-m_2^4-m_1^2p^2-m_2^2pq)q^2)}{m_1^2-m_2^2} \\
&+ ((m_1^2-m_2^2+p^2)pq-p^2q^2)f(p^2,m_1,m_2) \\
&+ (-m_1^2+m_2^2-p^2+pq)q^2f(q^2,m_1,m_1) \\
&+ (2(pq)^2+(m_1^2-m_2^2)q^2-pq(m_1^2-m_2^2+p^2+q^2))f((p-q)^2,m_1,m_2) \\
&+ (4m_1^2(pq)^2-2(m_1^2-m_2^2+p^2)pqq^2+q^2((m_1^2-m_2^2)^2+(p^2)^2 \\
&+ p^2(-2(m_1^2+m_2^2)+q^2)))c_0). \qquad \text{(B.17)}
\end{aligned}
$$

Final results for our loop calculations were greatly simplified due to expansion in small lepton masses (or rather in the ratio m_l/M_W). However, such expansion is not trivial - lepton masses appear both as squares of external loop momenta and as the loop particle masses. Thus, expansion has to be done both in all relevant arguments. In addition, the algebraic expressions for tensor functions formally contain inverse powers of external momenta ($1/q^2$ in case of b-functions and $1/(p^2q^2-(pq)^2)$ for c-functions). To get finite limit $p,q\to 0$ one needs also to expand b_0, c_0 functions to appropriate order in external momenta and cancel the leading terms up to some power of momenta in the numerator higher then momenta in the denominator. For our purposes it was sufficient to expand b_0 up to 6-th order in momenta and and c_0 up to 4th order. The expansion of b_0 reads as:

$$
\begin{aligned}
b_0(p,m_1,m_2) &= \frac{2}{4-d}+1-\log\frac{m_1m_2}{\mu^2}+\frac{(m_1^2+m_2^2)\log\frac{m_2}{m_1}}{m_1^2-m_2^2} \\
&+ p^2f_1(m_1,m_2)+(p^2)^2f_2(m_1,m_2)+(p^2)^3f_3(m_1,m_2), \qquad \text{(B.18)}
\end{aligned}
$$

where

$$
\begin{aligned}
f_1(m_1,m_2) &= \frac{m_1^2+m_2^2}{2\left(m_1^2-m_2^2\right)^2}-\frac{2m_1^2m_2^2\log\frac{m_1}{m_2}}{\left(m_1^2-m_2^2\right)^3},\\
f_2(m_1,m_2) &= \frac{m_1^4+10m_1^2m_2^2+m_2^4}{6\left(m_1^2-m_2^2\right)^4}-\frac{2m_1^2m_2^2\left(m_1^2+m_2^2\right)\log\frac{m_1}{m_2}}{\left(m_1^2-m_2^2\right)^5},\\
f_3(m_1,m_2) &= \frac{m_1^6+29m_1^4m_2^2+29m_1^2m_2^4+m_2^6}{12\left(m_1^2-m_2^2\right)^6}-\frac{2m_1^2m_2^2\left(m_1^4+3m_1^2m_2^2+m_2^4\right)\log\frac{m_1}{m_2}}{(m_1^2-m_2^2)^7}.
\end{aligned}
\tag{B.19}
$$

For c_0 expansion we assume two mass arguments to be equal as we considered only flavor-diagonal couplings to photon:

$$
\begin{aligned}
c_0(q,p,m_1,m_1,m_2) &= cp_{00}(m_1,m_2)+cp_{02}(m_1,m_2)p^2+cp_{11}(m_1,m_2)pq\\
&+ cp_{20}(m_1,m_2)q^2\\
&+ 3cp_{04}(m_1,m_2)(p^2)^2+3cp_{13}(m_1,m_2)p^2pq\\
&+ cp_{22a}(m_1,m_2)(2pq^2+p^2q^2)\\
&+ cp_{22b}(m_1,m_2)p^2q^2+3cp_{31}(m_1,m_2)q^2pq+3cp_{40}(m_1,m_2)(q^2)^2,
\end{aligned}
\tag{B.20}
$$

where

$$
\begin{aligned}
cp_{00}(m_1,m_2) &= -\frac{1}{m_1^2-m_2^2}-\frac{2m_2^2\log\frac{m_2}{m_1}}{\left(m_1^2-m_2^2\right)^2},\\
cp_{02}(m_1,m_2) &= -\frac{m_1^2+5m_2^2}{2\left(m_1^2-m_2^2\right)^3}-\frac{2m_2^2\left(2m_1^2+m_2^2\right)\log\frac{m_2}{m_1}}{\left(m_1^2-m_2^2\right)^4},\\
cp_{11}(m_1,m_2) &= \frac{m_1^2+5m_2^2}{2\left(m_1^2-m_2^2\right)^3}+\frac{2m_2^2\left(2m_1^2+m_2^2\right)\log\frac{m_2}{m_1}}{\left(m_1^2-m_2^2\right)^4},\\
cp_{20}(m_1,m_2) &= \frac{-2m_1^4-5m_1^2m_2^2+m_2^4}{6m_1^2\left(m_1^2-m_2^2\right)^3}-\frac{2m_1^2m_2^2\log\frac{m_2}{m_1}}{\left(m_1^2-m_2^2\right)^4},\\
cp_{04}(m_1,m_2) &= -\frac{m_1^4+19m_1^2m_2^2+10m_2^4}{9\left(m_1^2-m_2^2\right)^5}-\frac{2m_2^2\left(3m_1^4+6m_1^2m_2^2+m_2^4\right)\log\frac{m_2}{m_1}}{3\left(m_1^2-m_2^2\right)^6},\\
cp_{13}(m_1,m_2) &= \frac{2\left(m_1^4+19m_1^2m_2^2+10m_2^4\right)}{9\left(m_1^2-m_2^2\right)^5}+\frac{4m_2^2\left(3m_1^4+6m_1^2m_2^2+m_2^4\right)\log\frac{m_2}{m_1}}{3\left(m_1^2-m_2^2\right)^6},\\
cp_{22a}(m_1,m_2) &= -\frac{2\left(m_1^4+19m_1^2m_2^2+10m_2^4\right)}{9\left(m_1^2-m_2^2\right)^5}-\frac{4m_2^2\left(3m_1^4+6m_1^2m_2^2+m_2^4\right)\log\frac{m_2}{m_1}}{3\left(m_1^2-m_2^2\right)^6},\\
cp_{22b}(m_1,m_2) &= \frac{2m_2^2\left(m_1^2+m_2^2\right)\log\frac{m_2}{m_1}}{\left(m_2^2-m_1^2\right)^5}-\frac{m_1^4+10m_1^2m_2^2+m_2^4}{6m_1^2\left(m_1^2-m_2^2\right)^4},
\end{aligned}
\tag{B.21}
$$

$$
\begin{aligned}
cp_{31}(m_1, m_2) &= \frac{4m_1^2 m_2^2 \left(2m_1^2 + 3m_2^2\right) \log \frac{m_2}{m_1}}{3\left(m_1^2 - m_2^2\right)^6} + \frac{3m_1^6 + 47m_1^4 m_2^2 + 11m_1^2 m_2^4 - m_2^6}{18m_1^2 \left(m_1^2 - m_2^2\right)^5}, \\
cp_{40}(m_1, m_2) &= \frac{-9m_1^8 - 104m_1^6 m_2^2 - 4m_1^4 m_2^4 - 4m_1^2 m_2^6 + m_2^8}{180m_1^4 \left(m_1^2 - m_2^2\right)^5} \\
&- \frac{2m_1^2 m_2^2 \left(m_1^2 + m_2^2\right) \log \frac{m_2}{m_1}}{3\left(m_1^2 - m_2^2\right)^6}.
\end{aligned}
$$

Note the although formulae for f_i functions and for cp_{ij} functions look to have singularities, actually they are all finite in the limit $m_1 = m_2$.

Appendix C

Z-boson contribution to the effective lepton-photon vertex

In this Appendix as an example we show details of calculation Z-boson contribution to effective lepton-photon vertex - both the reducible self-energy contribution and the 1PI-irreducible triangle diagram.

C.1 Self energy contribution

We start from simpler calculation of the self-energy contribution.

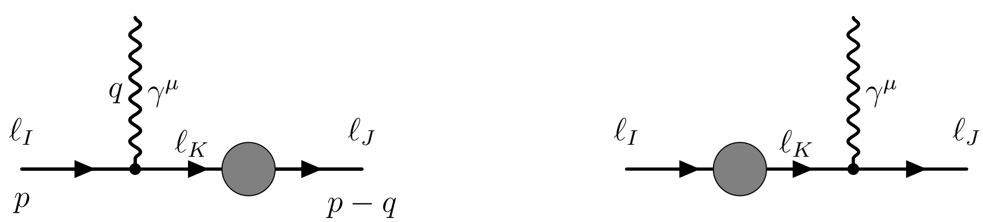


Figure C.1: Self energy diagrams contributing to $l^I \to l^J \gamma$ decay.

Let's decompose the lepton self energy in terms of scalar (S), vector (V), pseudo-scalar (P) and pseudo-vector (A) form factors as:

$$\Sigma^{IJ}(p) = \Sigma_V^{IJ} \not{p} + \Sigma_A^{IJ} \not{p} \gamma^5 + \Sigma_S^{IJ} + \Sigma_p^{IJ} \gamma^5. \tag{C.1}$$

Topologies of self-energy diagrams contributing to lepton-photon vertex are shown in Fig. C.1. We further denote the amplitude of the the left diagram as M_1 and the right diagram as M_2. Their respective amplitudes read as:

$$\begin{aligned} M_1 &= -i^3 e\bar{u}_J(p-q)\Sigma^{IJ}(p-q)\frac{\not{p}-\not{q}+m_I}{(p-q)^2-m_I^2}\gamma^\mu u_I(p)\epsilon^*_\mu(q), \\ M_2 &= -i^3 e\bar{u}_J(p-q)\gamma^\mu\frac{\not{p}+m_J}{p^2-m_J^2}\Sigma^{IJ}(p)u_I(p)\epsilon^*_\mu(q). \end{aligned} \tag{C.2}$$

If the external leptons are on-shell, one can use Dirac equation to reduce the amplitudes:

$$\bar{u}_J(p-q)(\not{p}-\not{q}) = m_J\bar{u}_J(p-q) \qquad \not{p}u_I(p) = m_I u_I(p). \tag{C.3}$$

Thus:

$$\begin{aligned} \bar{u}_J(p-q)\Sigma^{IJ}(p-q) &= \bar{u}_J(p-q)\left[m_J\Sigma_V^{IJ}(m_J^2)+\Sigma_S^{IJ}(m_J^2\right. \\ &+ \left.(m_J\Sigma_A^{IJ}(m_J^2)+\Sigma_P^{IJ}(m_J^2))\gamma^5\right] \\ &\equiv \bar{u}_J(p-q)(A^J+B^J\gamma_5), \\ \Sigma^{IJ}(p)u_I(p) &= \left[m_I\Sigma_V^{IJ}(m_I^2)+\Sigma_S^{IJ}(m_I^2)+\gamma^5(\Sigma_P^{IJ}(m_I^2)-m_I\Sigma_A^{IJ}(m_I^2))\right]u_I(p) \\ &\equiv (A^I+B^I\gamma_5)u_I(p). \end{aligned} \tag{C.4}$$

Using the notation introduced in eq. (C.4), the sum of diagrams can be simplified to the form

$$M_1+M_2 = ie\bar{u}_J[\frac{A^I-A^J}{m_I-m_J}\gamma^\mu - \frac{B^I-B^J}{m_I+m_J}\gamma^\mu\gamma^5]u_I(p)\epsilon^*_\mu(q). \tag{C.5}$$

As can be seen from eq. (C.5), self energy diagram can contribute only to vector form factors F_{VL} and F_{VR} in the effective lepton-photon vertex. Thus, they do not contribute directly to the expression for the radiative lepton decay rare $Br(l \to l'\gamma)$, but are necessary to cancel relevant part of other diagrams in order to make the total amplitude gauge invariant. Also, such vector form factors could contribute to 3-body charged lepton decays, if they are considered at the 1-loop level.

To finish this part our example we calculate explicitly the self-energy diagram shown in Fig. C.2. In terms of standard 2-point loop functions defined in Appendix B the expression for Σ can be written as:

$$\begin{aligned} -i\Sigma^{IJ} &= -i^4\sum_{K=1}^{3}\int\frac{d^dk}{(2\pi)^4}\frac{\gamma^\alpha(a^{KJ*}P_L+b^{KJ*}P_R)(\not{k}+m_K)\gamma_\alpha(a^{IK}P_L+b^{IK}P_R)}{(k^2-m_K^2)((k-p)^2-m_Z^2)} \\ &= \frac{2i}{4\pi^2}\sum_{K=1}^{3}\big(\not{p}(a^{IK}a^{KJ*}P_L+b^{IK}b^{KJ*}P_R)b_1(p,m_K,m_Z) \\ &- 2m_K(a^{IK}b^{KJ*}P_L+a^{KJ*}b^{IK}P_R)b_0(p,m_K,m_Z)\big)\,, \end{aligned} \tag{C.6}$$

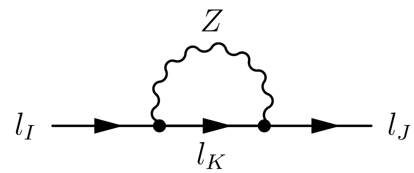


Figure C.2: Lepton self-energy diagram with Z boson exchange.

where the Z-lepton coupling constants a, b are defined in terms of Wilson coefficients in Feynman rules given in Appendix A.

Thus, the Z boson contribution to vector and scalar form factors of lepton self-energy read as:

$$
\begin{aligned}
\Sigma_V &= -\frac{1}{4\pi^2}\sum_{K=1}^{3}(a^{IK}a^{KJ*} + b^{IK}b^{KJ*})b_1(p, m_K, m_Z), \\
\Sigma_A &= \frac{1}{4\pi^2}\sum_{K=1}^{3}(a^{IK}a^{KJ*} - b^{IK}b^{KJ*})b_1(p, m_K, m_Z) \\
\Sigma_S &= \frac{2}{4\pi^2}\sum_{K=1}^{3}m_K(a^{IK}b^{KJ*} + a^{KJ*}b^{IK})b_0(p, m_K, m_Z), \\
\Sigma_P &= -\frac{2}{4\pi^2}\sum_{K=1}^{3}m_K(a^{IK}b^{KJ*} - a^{KJ*}b^{IK})b_0(p, m_K, m_Z).
\end{aligned} \tag{C.7}
$$

C.2 1PI irreducible triangle diagram

The calculation of the 1PI irreducible Z-boson contribution to effective lepton-photon vertex is more involved.

The general form of vertices $1, 2, 3$ in the triangle Z diagram displayed in Fig. C.3 can be written as:

1: $i\gamma^\alpha(a^{IK}P_L + b^{IK}P_R) \equiv i\gamma^\alpha(aP_L + bP_R)$
2: $i\gamma^\beta(a^{JK*}P_L + b^{JK*}P_R) \equiv i\gamma^\beta(a^*P_L + b^*P_R)$
3: $ie\gamma^\mu$
where to compactify the notation we temporarily do not display explicitly the flavor indices I, J, K of a, b couplings.

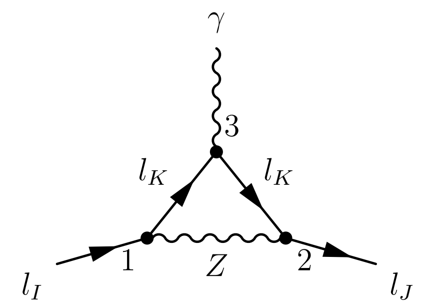


Figure C.3: 1PI irreducible Z-lepton contribution to lepton-photon vertex.

We can now write an expression for the amplitude associated with this diagram as:

$$M = -e\bar{u_J}(p-q)\int \frac{d^4k}{(2\pi^4}\frac{\gamma_\alpha(a^*P_L+b^*P_R)(\not{k}-\not{q}+m_K)\gamma^\mu(\not{k}+m_K)\gamma^\alpha(aP_L+bP_R)}{(k^2-m_K^2)[(k-q)^2-m_K^2][(k-p)^2-M_z^2]} \\ u_I(p)\epsilon^*_\mu(q). \tag{C.8}$$

To obtain the integral in the form of Passarino-Veltman decomposition, as given in Appendix B, we simplify first the numerator of the integrand:

$$\begin{aligned} Num &= \bar{u_J}(p-q)(a^*P_R+b^*P_L)\gamma_\alpha(\not{k}-\not{q}+m_K)\gamma^\mu(\not{k}+m_K)\gamma^\alpha(aP_L+bP_R)u_I(p) \\ &\equiv \bar{u_J}(p-q)(a^*P_R+b^*P_L)X(aP_L+bP_R)u_I(p), \end{aligned} \tag{C.9}$$

where X is defined as:

$$\begin{aligned} X &= \gamma_\alpha(\not{k}-\not{q}+m_K)\gamma^\mu(\not{k}+m_K)\gamma^\alpha \\ &= 2(k^2-m_K^2-2kq)\gamma^\mu+8m_Kk^\mu-4k^\mu\not{k}-4m_Kq^\mu+4k^\mu\not{q}+2\gamma^\mu\not{q}\not{k}. \end{aligned} \tag{C.10}$$

The full expansion of RHS in eq. (C.9) leads to a lengthy expression, so we split it into several terms differing by number of Dirac matrices and powers of loop momentum k. We define:

$$Num = A+B+C+D+E, \tag{C.11}$$

where the symbols on the RHS are:

$$
\begin{aligned}
A &\equiv \bar{u_J}(p-q)(a^*P_R+b^*P_L)2(k^2-m_K^2-2kq)\gamma^\mu(aP_L+bP_R)u_I(p) \\
&= 2(k^2-m_K^2-2kq)\bar{u_J}(p-q)\gamma^\mu(aa^*P_L+bb^*P_R)u_I(p), \\
B &\equiv \bar{u_J}(p-q)(a^*P_R+b^*P_L)(8m_Kk^\mu-4m_Kq^\mu)(aP_L+bP_R)u_I(p) \\
&= 4m_K(2k^\mu-q^\mu)(ab^*P_L+ba^*P_R)u_I(p), \\
C &\equiv -4k^\mu\bar{u_J}(p-q)(a^*P_R+b^*P_L)\not{k}(aP_L+bP_R)u_I(p) \\
&= -4k^\mu\bar{u_J}(p-q)\not{k}(aa^*P_L+bb^*P_R)u_I(p), \\
D &\equiv 4k^\mu\bar{u_J}(p-q)\not{q}(aa^*P_L+bb^*P_R)u_I(p) \\
&= -4m_Jk^\mu\bar{u_J}(p-q)(aa^*P_L+bb^*P_R)u_I(p) \\
&+ 4m_Ik^\mu\bar{u_J}(p-q)(aa^*P_R+bb^*P_L)u_I(p), \\
E &\equiv 2m_I^2\bar{u_J}(p-q)\gamma^\mu(aa^*P_L+bb^*P_R)u_I(p) \\
&= -4m_I(p^\mu-q^\mu)\bar{u_J}(p-q)(aa^*P_R+bb^*P_L)u_I(p) \\
&+ 2m_Im_J\bar{u_J}(p-q)\gamma^\mu(aa^*P_R+bb^*P_L)u_I(p). \qquad \text{(C.12)}
\end{aligned}
$$

Evaluating the integrals let us define

$$
Q' \equiv \int \frac{d^4k}{(2\pi^4)} \frac{-eQ}{(k^2-m_K^2)[(k-q)^2-m_K^2][(k-p)^2-M_Z^2]}, \qquad \text{(C.13)}
$$

where Q can be any of $A \ldots E$ quantities defined above. Then one has:

$$
\begin{aligned}
A' &= \frac{-2ie}{(4\pi)^2}[q^2(c_{21}-2c_{11}+c_{23}-c_{12})-m_K^2c_0+4c_{24} \\
&+ m_I^2(c_{22}+c_{23}-c_{12})-m_J^2(c_{23}-c_{12})] \\
&\times \bar{u_J}(p-q)\gamma^\mu(aa^*P_L+bb^*P_R)u_I(p), \\
B' &= \frac{-ie4m_K}{(4\pi)^2}[(2c_{11}-c_0)q^\mu+2p^\mu c_{12}] \times \bar{u_J}(p-q)(ab^*P_L+ba^*P_R)u_I(p),
\end{aligned}
$$

$$
\begin{aligned}
C' &= \frac{4iem_I}{(4\pi)^2}[(c_{21}+c_{23})q^\mu + (c_{22}+c_{23})p^\mu)]\bar{u_J}(p-q)(aa^*P_R + bb^*P_L)u_I(p) \\
&- \frac{4iem_J}{(4\pi)^2}[c_{21}q^\mu + c_{23}p^\mu]\bar{u_J}(p-q)(aa^*P_L + bb^*P_R)u_I(p) \\
&+ \frac{4ie}{(4\pi)^2}c_{24}\bar{u_J}(p-q)\gamma^\mu(aa^*P_L + bb^*P_R)u_I(p), \\
D' &= \frac{-4ie}{(4\pi)^2}(q^\mu c_{11} + p^\mu c_{12})[-m_J\bar{u_J}(p-q)(aa^*P_L + bb^*P_R)u_I(p) \\
&+ m_I\bar{u_J}(p-q)(aa^*P_R + bb^*P_L)u_I(p)], \\
E' &= \frac{-2ie}{(4\pi)^2}(q^2c_{11} + m_I^2c_{12})\bar{u_J}(p-q)\gamma^\mu(aa^*P_L + bb^*P_R)u_I(p) \\
&+ \frac{4ie}{(4\pi)^2}m_I(p-q)^\mu c_{12}\bar{u_J}(p-q)(aa^*P_R + bb^*P_L)u_I(p) \\
&- \frac{2ie}{(4\pi)^2}m_Im_Jc_{12}\bar{u_J}(p-q)\gamma^\mu(aa^*P_R + bb^*P_L)u_I(p), \qquad \text{(C.14)}
\end{aligned}
$$

and the arguments of c_{ij} functions are always understood as $c_{ij} \equiv c_{ij}(q, p, m_K, m_K, M_Z)$.

In order to transform the result to the form of eq. (2.20), one needs to use also the appropriate Gordon identities to remove explicit p^μ dependence:

$$
\begin{aligned}
p^\mu\bar{u_J}(p-q)P_Lu_I(p) &= \frac{1}{2}q^\mu\bar{u_J}(p-q)P_Lu_I(p) + \frac{1}{2}\bar{u_J}(p-q)\sigma^{\mu\nu}q^\nu P_Lu_I(p) \\
&+ \frac{i}{2}q^\mu\bar{u_J}(p-q)(P_Lm_J + P_Rm_I)u_I(p), \\
p^\mu\bar{u_J}(p-q)P_Ru_I(p) &= \frac{1}{2}q^\mu\bar{u_J}(p-q)P_Ru_I(p) + \frac{1}{2}\bar{u_J}(p-q)\sigma^{\mu\nu}q^\nu P_Ru_I(p) \\
&+ \frac{i}{2}q^\mu\bar{u_J}(p-q)(P_Rm_J + P_Lm_I)u_I(p). \qquad \text{(C.15)}
\end{aligned}
$$

The final result for Zll 1-loop 1PI irreducible contribution to lepton-photon coupling reads as:

$$
\begin{aligned}
F_{VL}^{1PI-Zll} &= \frac{2e}{4\pi^2}(-aa^*m_K^2c_0 - aa^*q^2c_{11} + (2a^*bm_Im_K + 2ab^*m_Jm_K - aa^*q^2)c_{12} \\
&+ aa^*q^2c_{21} - bb^*m_Im_Jc_{22} + aa^*q^2c_{23} + 2aa^*c_{24}), \\
F_{VR}^{1PI-Zll} &= \frac{2e}{4\pi^2}(-bb^*m_K^2c_0 - bb^*q^2c_{11} + (2ab^*m_Im_K + 2a^*bm_Jm_K - bb^*q^2)c_{12} \\
&+ bb^*q^2c_{21} - aa^*m_Im_Jc_{22} + bb^*q^2c_{23} + 2bb^*c_{24}),
\end{aligned}
$$

$$
\begin{aligned}
F_{TL}^{1PI-Zll} &= -\frac{2e}{4\pi^2}(a(a^*m_J - 2b^*m_K)c_{12} + bb^*m_I c_{22} + (bb^*m_I - aa^*m_J)c_{23}) \\
F_{TL}^{1PI-Zll} &= -\frac{2e}{4\pi^2}(b(b^*m_J - 2a^*m_K)c_{12} + aa^*m_I c_{22} + (aa^*m_I - bb^*m_J)c_{23}), \\
F_{SL}^{1PI-Zll} &= \frac{-2e}{4\pi^2}(2ab^*m_K c_0 - 2(bb^*m_I - aa^*m_J + 2ab^*m_K)c_{11} \\
&+ (aa^*m_J - 2bb^*m_I - 2ab^*m_K)c_{12} \\
&+ 2(bb^*m_I - aa^*m_J)c_{21} + bb^*m_I c_{22} + (3bb^*m_I - aa^*m_J)c_{23}), \\
F_{SL}^{1PI-Zll} &= \frac{-2e}{4\pi^2}(2a^*bm_K c_0 - 2(aa^*m_I - bb^*m_J + 2a^*bm_K)c_{11} \\
&+ (bb^*m_J - 2aa^*m_I - 2a^*bm_K)c_{12} \\
&+ 2(aa^*m_I - bb^*m_J)c_{21} + aa^*m_I c_{22} + (3aa^*m_I - bb^*m_J)c_{23}). \qquad \text{(C.16)}
\end{aligned}
$$

Further calculation is rather tedious and has been done with the use of Mathematica program. In the first step, tensor integrals c_{ij} must be reduced to combinations of master integrals a_0, b_0, c_0. Later, one needs to add 1PI-irreducible and self-energy type contributions from the Z boson exchanges. At this stage one can check exactly, without any approximations or expansions (which is an important test if the calculations has been done correctly) that the vector form factors F_{VL}, F_{VR} vanish exactly for $I \neq J$ if $q^2 \to 0$, as required by gauge invariance.

Using of full general version of c_0 function (which contain dilogarithms, may have complex phase etc.) is not necessary as the lepton masses are very small compared to Z boson mass. Thus, one can perform expansion in momenta of external particles, using the formulae given in Appendix B. Finally, yet another expansion can be done in the ratio of loop particle masses, in this case m_K/M_Z. After all those approximation result given in eq. (C.16) can be reduced to a simple analytical form given in Table 4.2.

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